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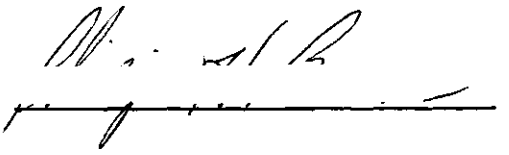

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DECISION TECHNIQUES FOR A STOCK MARKET HEDGE SITUATION

A THESIS

Presented to

The Faculty of the Division of Graduate

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By

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DECISION TECHNIQUES FOR A STOCK MARKET HEDGE SITUATION

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SUMMARY

The purpose of this thesis is to develop a set of decision rules to apply to a stock market hedge situation. A stock market hedge situation is one in which an investor sells short warrants of a company and at yhe same time purchases common stock long in the same company. The ultimate goal of these decision rules is to allow the investor to commit to a position that will insure he has no loss on his investment over a predetermined common stock price range. Decision rules are developed for two specific processes. The first is the single decision process in which the potential hedge situations are determined, their future profit potential evaluated, and the selection of one situation from the group of potential situations is accomplished. The second specific process is the dynamic multi-decision process. In this process periodic evaluation of the selected situation is accomplished based upon new information pertaining to this investment. Depending upon this reevaluation appropriate adjustments to the selected situation are accomplished so as to insure profitability at the future horizon.

Extensive use, throughout the development of the decision rules, is made of a profit graph which visually portrays the various positions that will be most profitable in the determined common stock price range. Other important procedures used include assessment of probability distributions over common stock price ranges and expected value decision making techniques. The development of this thesis is oriented toward

practical application with ease of computation, ease of understanding, and minimal time requirements as primary considerations. Illustrative examples are included to demonstrate application of the developed decision rules. The results of this thesis indicate that the profit graph is a valuable aid in this type of decision making; that application of the decision rules results in substantial profits; and that the decision rules applied to the hedge situation satisfy general investor objectives.

GLOSSARY

Warrants

Stock purchase warrants are options to purchase common stock of a company at a stipulated price during a specific time period. Normally, one warrant plus the stipulated amount of money (exercise price) allows the warrant holder to purchase a fixed number of shares of common stock. The number of shares of stock to be purchased, the exercise price, and the future date of warrant expiration are set by the issuing company. Warrants are traded on the various stock exchanges and on the over-the-counter market in the same manner as common stock. They pay no dividend and normally have an expiration date after which time they become worthless; although some warrants have no expiration date and are said to be perpetual.

Short Sales

A short sale is the sale of stock (or warrants) that the seller does not own but which he expects to acquire in the future. A short sale is normally made in anticipation of a decline in the price of the stock. The seller hopes to sell the stock now and buy it back later at a lower price, thus making a profit on the transaction. The broker to whom the order is given arranges to borrow the stock from his own resources or from some other broker who has stock available for lending. In most cases, short sellers can hold their position until they decide to "cover" their position by buying the appropriate number of shares and

delivering them to the lender. To preclude depressing the stock market with short sales, most exchanges require that a short sale be designated as such when placed, and it cannot be executed at a price lower than the last preceding regular (as distinguished from short) sale.

Margin

The two main types of accounts with brokers are cash accounts and margin accounts. In a cash account, purchases are made outright for full ownership and sales are made against immediate delivery of securities. In a margin account the buyer places in his account cash or securities equal to a portion of the price of the security (the margin, currently 55% initially) and the broker advances the balance at an appropriate interest rate (currently approximately 6%). If the price of the security declines, the buyer must keep the margin good by depositing additional cash or securities so that the margin will meet the required minimum percentage (currently approximately 30%) of the market price.

Fundamental Analysis

Fundamental analysis is a type of security analysis that considers the industry, the company, its management, its balance sheet and income statement, its earning progress, its products, its future earnings outlook, etc., in arriving at conclusions as to commitments to be made. The largest portion of investors today use this type of analysis.

Technical Analysis

Technical analysis is a type of security analysis that deals primarily with a stock's own price movement and volume of sales action as

interpreted either through charts or the ticker tape. This type of analysis, when used alone, does not consider the fundamental factors at all, but instead follows the concept that the stock's own action will predict its future direction.

Hedge Situation

A stock market hedge situation is one in which an investor sells short warrants of a company and at the same time purchases common stock long in the same company.

CHAPTER I

INTRODUCTION

Background

The realm of investments is probably one of the most widely studied fields in existence today. Considerable work and study has been done in all types of investment media to include: insurance and retirement investments; deposit type investments such as savings and time deposits in banks; securities investments; real estate investments; and direct investment in business properties. This list is by no means all inclusive but does indicate the diversity of investments.

The area of securities investments is a field that is constantly undergoing change as investors, traders, and speculators attempt to bargain and trade for those securities that they feel will produce profit for them. Investors range from the "little man" or individual small investor to the large institutions with each attempting to meet his own financial objectives.

The list of securities that investors may choose from is impressive. It includes common stock, preferred stock, corporate bonds, municipal bonds, options such as puts, calls, rights, etc., and warrants, to name a few. These securities are traded on the New York Stock Exchange, American Stock Exchange, various regional stock exchanges, and in the over-the-counter market.

In recent years, the number of small investors has drastically increased. Many of these investors participate directly in the stock

market, but a very large proportion of these investors have elected to invest through open and closed end investment companies. One of the primary reasons for the large scale move to the mutual funds is the fact that the stock market is a highly competitive arena, and the small investor often feels that the professional financial management of the mutual funds will eliminate much of the research time requirement and hopefully produce better results. This is a debatable point that may or may not be true. In any case, many mutual funds have performed in an outstanding manner over the past decade while many others have performed very poorly.

The primary investment means for most investors remains direct participation in the stock market by buying securities long or selling securities short. The stock market hedge situation, a direct investment in the stock market, may also be worthy of consideration by investors. The hedge situation can be researched and committed to without excessive time requirements. When comparing the hedge situation to actually buying stock in a company and holding it for future appreciation, it appears that the hedge situation reduces the risk on the investment because it normally increases the price range over which the investment will be profitable. This is shown by Figure 1 and its accompanying explanation. It is also possible that the hedge situation carries less risk than investment in most growth type mutual funds. This is a much more difficult comparison since one of the strong points in favor of mutual funds is their diversification of investments; however, mutual funds generally purchase common stock long. Again, the price range of profitability is normally less in this case than if a hedge position could be taken.

Since the stock market hedge situation is the basic system

investigated in this study, it is important that such an investment situation is understood. A company issues warrants for various financial reasons. The underlying reasons are virtually unimportant for the investor in a hedge situation as he only needs to know the three basic terms of the warrant. First, he must know the expiration date of the warrant. This is the last date it can be converted into common stock. Secondly, he must know the exercise price of the warrant. This is the amount of money required to convert the warrant to the prescribed number of shares of common stock. Third, he must know the number of common stock shares into which each warrant can be converted. All of this information is readily available as will be explained later.

With the above information, the theoretical value of the warrant can then be determined as follows:

Theoretical Value of Warrant at any time t	=	Current Price of Common Stock at time t	X	Number of Common Shares into which Warrant can Convert	-	Conversion Price of Warrant	(1)

If the warrant is selling at a price greater than the theoretical value then the warrant is said to be overpriced. This is the type of situation the hedge investor is looking for. Rarely will warrants trade for less than their theoretical value because when this occurs, arbitrage takes place as the floor traders, who pay no commissions, purchase the underpriced warrants, immediately convert them to common stock, and make a quick profit.

One of the basic assumptions for the hedge situation is that as a warrant nears expiration its actual price approaches its theoretical value. At expiration, the actual value of the warrant is, for all

practical purposes, its theoretical value. While this statement could undoubtedly be proven by a comprehensive study of warrant price action, it will be treated as an assumption in this study.

The intriguing aspect of the hedge situation is that it is possible to always obtain a profit from commitment to this type of situation as long as the price of the common stock remains in a predetermined range. The common stock price may move up or down, but as long as it remains in the range, up until the time the warrant expires, no loss will result. The reason for this is that the price of the warrant, particularly as it nears expiration, is dependent upon the price of the common stock. The problem facing the investor is to determine the ratio of short warrants to long common stock in which he should invest. Figure 1 is a profit graph showing a general sample of profit curves for various regular and hedge positions. This illustrates the range of common stock prices that will yield a profit when the warrant expires.

The profit for each hedge ratio, in which the number of short warrants is greater than or equal to the number of shares of long common stock, is maximized if the common stock is selling at the adjusted conversion price when the warrant expires. The concept of the adjusted conversion price is explained fully in Chapter III. The reason for this maximization is because at the adjusted conversion price the theoretical value of the warrant is zero yet any loss on the common stock is minimal. Therefore, maximum gain on the short warrant occurs, coupled with minimal loss, and sometimes gain, on the common stock. For example, assume a hedge situation was being investigated in which a warrant was convertible into one common stock share at $10. Suppose at the time of investigation

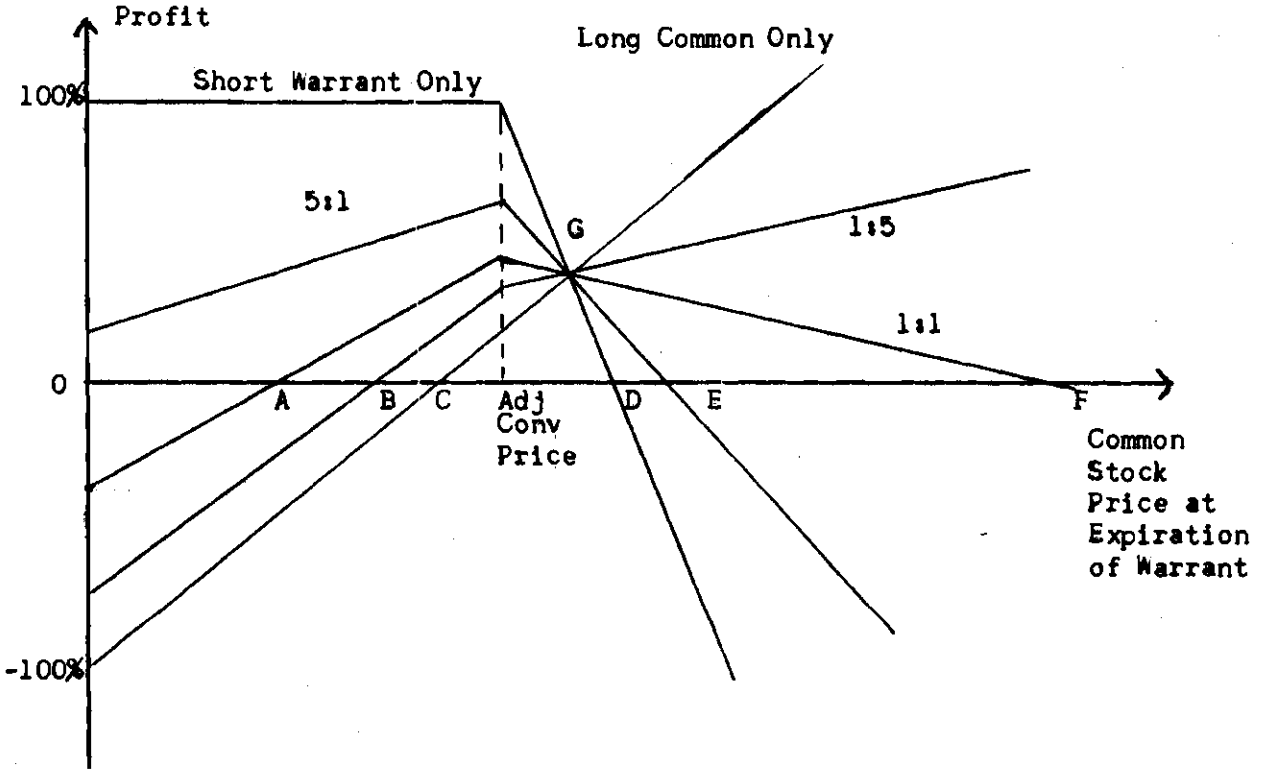

Figure 1. General Profit Curves for Various Positions

the common stock was selling at $9 per share, the warrants at $5 each, and the hedge ratio considered was 2:1. The adjusted conversion price is $10. If the common stock is selling at $10 per share when the warrant expires, the theoretical value of the warrant, from Equation (1), is (10(1) - 10), or zero. Therefore, the gain on the 2 short warrants is 2(5 - 0) =10, and the gain on the common stock is 10 - 9 = 1. The total gain on the investment then is $11. Now consider the common stock at $9 per share when the warrant expires. The theoretical warrant value is 9(1) - 10, or zero since the warrant cannot have a negative value. Thus there is no gain on the common stock but there is a $10 gain on the short warrants giving a total profit of $10. Finally, consider the common stock selling

at $11 per share when the warrant expires. In this case, the theoretical value of the warrant is 11(1) - 10 = 1 resulting in a gain of 2(5 - 1) = 8 on the short warrants. The gain on the common stock is 11 - 9 = 2 giving a total profit of $10. From this example, profits are maximized at the adjusted conversion price. This is a relatively simple example but it demonstrates this point. This maximization at the adjusted conversion price is true for all cases in which the number of short warrants is equal to or greater than the number of shares of long common stock.

It can be seen from Figure 1 that a profit exists for a short warrant only position as long as the common stock is selling in the price range O-D when the warrant expires. Similarly, profit exists for ratios of 1:1 and 5:1 if the common stock is selling in the price range A-F and O-E respectively when the warrant expires. Profit exists for a long common only position if the common sells at a point greater than C. Profit similarly exists for a 1:5 ratio if the common stock is selling at a price greater than B when the warrant expires. The points of intersection on the common stock price axis are arbitrary for this general example. In an actual evaluation these points can be specifically determined as is shown in Chapter III.

The profit curves described are merely representative of an entire family of curves. For hedge ratios greater than 5:1 the profit curves will move toward the short warrant only profit curve as a limit. For ratios less than 5:1 the profit curve will move toward the 1:1 ratio profit curve. For ratios less than 1:5 (e.g. 1:6, 1:7, etc.) the profit curves will tend to approach the long common only profit curve. For ratios greater than 1:5 (e.g. 1:4, 1:3, etc.) the profit curves will move

toward the 1:1 ratio profit curve. It is interesting to note that at point G on Figure 1 all profit curves intersect. This occurs in all cases when plotting this type of profit graph, but is generally not significant. More is explained about this intersection point, to include procedures for calculating this point, in Chapter III. Procedures for constructing the various profit curves will be presented in detail in Chapter III.

An important point to remember about the hedge situation is that the warrant will not be converted to common stock. Instead, it will be traded as a separate entity. A day or two before the expiration date of the warrant the entire hedge situation should be closed out.

With this general background of the stock market hedge situation, it is obvious that this situation offers many possibilities for gain using securities, specifically common stocks and warrants, as an investment vehicle. The most difficult problem to be solved is to determine a method for selecting ratios of short warrants to long common stock that will ensure profitability at the future expiration of the warrant. Chapters III and IV will include comprehensive development of procedures and rules to accomplish this, and at the same time, magnify the scope of the general discussion presented here and in the glossary.

Purpose

The purpose of this research is to develop a set of decision rules to apply to a stock market hedge situation. The ultimate goal of these rules is to allow the investor to commit to a position that will insure that he has no loss on his investment over a predetermined

common stock price range.

Specifically, the research is directed to the development of decision rules for the following two problems:

1. The single decision problem: This segment of the study deals with the choice of potential hedge situations, the evaluation of the future profit potential of these situations, and the ultimate selection of one hedge situation from the group of potential situations.

2. The dynamic multi-decision problem: This problem considers periodic evaluation of the selected hedge situation based upon new information pertaining to this investment. Depending upon the reevaluation, appropriate updating and adjustment of the selected hedge situation will be accomplished so as to insure profitability at the future horizon.

Scope of the Study

This research is oriented to solving the two specific problems previously stated. In developing decision rules, fundamental analytical methodology are incorporated. Other heuristic optimization techniques are incorporated and explained throughout.

The underlying assumptions of the study, stated again for convenience, are as follows:

1. The price action of a warrant is dependent upon the price action of the common stock, particularly as the warrant nears expiration.

2. The price of a warrant at expiration will be its theoretical value.

The development of rules and procedures is oriented toward practical application. Ease of computation, time considerations, and ease

of understanding are the points of primary emphasis. The goal is to insure that no net loss is taken on an investment. A reduction in profitability from the optimum is the tradeoff that is usually made to insure this profitability.

In selecting one hedge situation from a group of potential situations, no consideration is given to incorporating procedures for the "portfolio selection problem." While it would undoubtedly be possible to determine optimum strategies for allocating available cash to several of the potential hedge situations, it is not essential to the accomplishment of the purpose of this study.

All equations and computations involving future expected return are accomplished with the understanding that the investor is strictly using a cash account for his transactions. It is possible to incorporate the use of margin brokerage accounts, and this type of account should produce greater profits; however, for simplicity of development of the decision rules, margin accounts are not considered.

In the various evaluations made throughout this study, the brokerage commissions for purchase and sales of securities have not been included. The reason for this exclusion is to facilitate understanding of the rules and procedures developed. These commissions are relatively small and should not significantly alter any of the results obtained.

In the final analysis, the techniques and procedures developed for the various decision rules will require application by persons with a basic understanding of the stock market and mathematical probability theory.

Procedures

To accomplish the stated purpose of this research, a logical and systematic structure, following a general systems approach, is incorporated. This is to facilitate formulation of the steps and procedures inherent with the decision rules. The next chapter outlines the pertinent descriptive literature that has been written in the area of security analysis and methodology.

Chapter III presents the actual development of the single decision process, and includes the formulation of the specific decision rules that will be used in solving this basic problem. Chapter IV contains the development of the rules and procedures for the dynamic multi-decision process. Chapter V includes validation of the system model and decision rules by applying actual historical stock market data to the model and observing the results. Conclusions and recommendations are presented in Chapter VI.

CHAPTER II

LITERATURE SURVEY

Introduction

The literature survey presented in this chapter illustrates the basis for development of the stock market hedge situation system explained in the next chapter. Because of the vast amounts of literature written in some of the areas of interest of this study, attempts have been made to include and cite only the most pertinent works of the applicable literature. The works reviewed are meant to be representative and not necessarily exhaustive in most cases.

The research involved in this thesis probes into many different areas. These areas can be divided into two general categories. The first category is security analysis which generally includes technical and fundamental analysis, prediction of future stock market prices, statistical analysis techniques (including the portfolio selection problem), options, and warrants, and the hedge situation. The second general category is methodology which includes some of the analytical procedures actually selected for use in developing this hedge situation system. These analytical procedures consist of probability assessment and expected value theory and decision making.

In considering the security analysis aspect of the research the general format for presentation of the applicable literature will be chronological, from the classical work, to the present day work, and

finally to the current state of the art in this research area. In the category of methodology, the format for presentation will be to cite various representative works pertaining to the methodology actually used in developing this system.

Security Analysis Aspect of Research

The classical work in the field of security analysis is the development of the fundamental and technical approach to analyzing various stocks and companies. The work in fundamental analysis dates back to the formal formation of the stock exchange in this country in 1792 (30). While the stock market of this era was nothing as we know today, it did provide a means for speculators to trade in a limited number of securities and commodities. The principal technique used for analyzing available securities was to try to obtain inside information, on the various banks and companies, that was not yet publicly known (30). Men such as Duer and Clavier controlled much of this early speculation. Clavier described the early technique by instructing one of his associates to:

> Find a good situation....Study it, and if at first view it looks romantic, find the means of saving it from that objection; converse upon it with intelligent persons; find such as are sufficiently attached to great objects to be willing to concur in them with zeal, when they are designed for the aid and consolation of humanity. (30)

Throughout most of the 19th century, the stock market was virtually controlled by financial tycoons such as Cornelius Vanderbilt, J. P. Morgan, and others (30). It was not a common investment area for most people. The analysis techniques applied were basically founded upon earnings and projected earnings of the various companies (30). The analytical techniques were, however, almost completely overshadowed by the

widespread manipulations and pool operations prevalent during this era.

Around the turn of the century the work in technical analysis began to emerge. Men such as Richard D. Wyckoff (36) and Jesse L. Livermore (15) began to develop techniques for recording price and volume actions, and they applied these recorded actions to their investment decisions (15). Ultimately these records were transformed to vertical line charts and point and figure charts (36). The underlying concept of these pioneers was the idea that the stock market, by its own action, would predict its future direction (36). Others, in the early 20th century began applying geometric patterns to the various charts, and developed theories of future stock price direction based upon these patterns (36). The classical technique theories developed remain virtually unchanged today as more and more investors have subscribed to the technical viewpoint of security analysis. Authors such as Jiler and Edwards and Magee have amplified the early classical work making the technical approach more applicable to today's stock market (31).

Writings concerning fundamental analysis did not appear, to any degree, until the 1920's. The early years of the 20th century were much like the 19th century as far as the stock market was concerned. Control of the financial world remained with a powerful few; pool operations and manipulations were commonplace until the Securities Act of 1933 was passed (30). In the 1920's there was not a widespread move by the public, to the stock market, as is often portrayed when discussing events leading to the 1929 crash (30). In actuality, there were less than 1.6 million active accounts reported by brokerage firms in 1929 (30). Many of the "investors" during this period were gripped with speculative fever; almost

every stock was moving up in price. Selection could almost be made at random with resulting profits. Those investors applying fundamental techniques still based their emphasis on earnings in making investment decisions (30).

The early pioneers in the fundamental literature were men such as John Moody (23) and Jules I. Bogen (5) who published works outlining basic evaluation techniques. The basis of fundamental analysis was still earnings of companies but the area was expanded to include other factors such as industry grouping performance, industrial conditions, and the general economic conditions of the time (23). The present day work in this area follows the classical development and procedures except that the economic and industrial factors and considerations have expanded in scope and quantity as this country continued to grow and develop. Authors such as Rosenberg (26) and Loeb (16) have led the field in updating the fundamental factors; many of which are used in this thesis.

In general, the ultimate goal of both the technical and fundamental approaches to security analysis is to forecast or predict future stock prices. Another procedure has also evolved which attempts to predict future stock prices. This procedure is in the area of statistical analysis, a much more scientific approach. The classical work in this area was done by the French mathematician Bachelier (1) in 1900. He developed a theory which hypothesized that stock price changes, over periods of time, were independent (6). His work, based upon time series, eventually led to the random walk theory of stock market prices (10). In actuality, the random walk theory and the work concerning distributions

of stock price changes are interrelated in that the concept of distributions of stock price differences is contained as a segment of the random walk theory.

Proponents of the random walk theory maintain that price changes result from changes in the state of knowledge, which in turn results from new information. Since the flow of information is random in nature, the resulting changes in price must also be random (10). Hence, the level of prices is actually going through a random walk. This idea basically means that the problem of predicting a future price is the same as predicting the next price change. The critical aspect of this prediction is the time period over which the prediction is to be made (10).

Bachelier's classical work was largely ignored until 1934 when Holbrook Workings "rediscovered" the random walk theory and supported it with considerable empirical evidence (10). In 1953 Kendall obtained empirical results which further supported the random walk model (10).

In 1959 interest in the random walk theory was renewed. Cootner (7) edited a book which contains chronological works dealing with this theory. Expansion of the theory was accomplished by Granger and Morganstern (10) as they explored empirical work done on the stock market and its operations. The study in this area during the 1960's follows two general schools of thought related to the random walk theory. The first group agrees that sequences of prices follow a random walk with price changes normally distributed (6). The second group agrees that sequences of prices follow a random walk with price changes following a stable Paretian law with infinite variance (18). The stable Paretian

law is a Cauchy distribution with finite mean but infinite variance. These groups do disagree in their choice of appropriate probability distributions and/or in their choice of the appropriate parameters, time or transactions, that the distributions of stock price differences should be taken across (18).

Concerning the distribution of stock price differences, Brada, Ernst, and Van Tassel (18) argue convincingly that Bachelier's hypothesis of independent price changes over time periods should be modified to state that stock prices are independent when taken across transactions.

Of course, many do not agree with the random walk theory in part or in whole. Cootner and Baumol, as presented by Bartos (2), envision the path of stock prices, over long periods of time, as consisting of a number of trends, each of which is a random walk with reflecting barriers. Others disagree with the theory because of the relatively short time periods (one month or less) used in empirically developing this theory (10).

The 1960's brought forth considerable literature concerning all aspects of the stock market. One of the most important ideas regarding portfolio analysis was developed by Markowitz (20) in 1959. The Markowitz portfolio selection model is a model in which an optimal portfolio is defined as that which has the highest expected return for any given level of risk. Markowitz selects these portfolios by showing that an efficient portfolio consists of either a set of stocks which have a maximum expectation of return for a given variance in the return, or alternatively, a set of stocks which, for a fixed return, have a minimum variance in returns (20). Markowitz's work also played a large role in

the development of decision theory dealing with decision making under uncertainty.

Much of the 1960's literature deals with the Markowitz model Baumol (3) in 1963, modified the Markowitz model by presenting new criteria which reduced the set of efficient portfolios. Sharpe (27), also in 1963, presented a simplified diagonal model which reduced the number of estimates concerning probabilistic future performance of securities required for the Markowitz technique. Mao and Sarndal (19) used a Bayesian approach and reformulated the Markowitz model in 1967. McFarlane (21), in 1967, produced results of applying the Markowitz model to an actual securities investment program. Bartos (2), in an unpublished doctorial dissertation, developed procedures to obtain inputs to the Markowitz model. Up until this work, in 1969, Markowitz and other authors had assumed these inputs. From the preceding discussion of the representative works, it can be seen that the classical work by Markowitz has received considerable attention and modification in recent times.

Some of the other work in the 1960's was concerned with the options market (puts, calls, and straddles). Taylor (32) developed an optimal strategy for put and call holders under a random walk model in 1967. The most noteworthy of the work in the options area was done by Malkiel and Quant (17) in 1969. These authors developed a set of optimal strategies to be used when dealing in the options market. Their development involves the use of utility theory as well as decision making under uncertainty (17).

One area that has received very little attention in security analysis is that of warrant analysis and hedging. Warrants are relatively

new in the field of securities. The first known warrant, of American Power and Light Company, appeared in 1911. The first warrant listed on an exchange appeared in 1923 (33). The first written work concerning warrants was by Fried (9) in 1949. This work was informative in nature, merely describing warrants and illustrating the leverage potential in warrants. Fried has updated his pamphlet periodically, in connection with his warrant advisory service, and does discuss stock market hedging, although on an extremely elementary scale. Other major financial publications, such as Barrons, have periodically mentioned warrants but only in the context of having leverage potential. The only work known dealing explicitly with the stock market hedge situation is by Thorp and Kassouf (33). This work, published in 1967, details many procedures that can be used in investing in a hedge situation. Much of the explanation presented by these authors is informative in nature as their purpose in presenting the hedge situation investment is to convey the idea that it is possible to gain 25% per year through this specific type of investment. It is obvious that much research went into their book though no specific substantiated decision rules are presented. Most of their guidelines are general rules of thumb. Many of their ideas, particularly a form of the profit graph and the concept of the adjusted conversion price are used extensively in this thesis. Thorp and Kassouf consider the hedge situation primarily with warrants, but they do mention this type of investment using convertible securities very briefly. For the most part, these authors consider only hedge investments. They do not consider the long common stock only position or the short warrant only position in connection with their analysis techniques. The work by Thorp and Kassouf is

used as the basic concept for this study. This thesis must be considered to be amplification, extension, and modification of their work as it moves to an area that has not been previously covered. Specifically, to the logical, detailed development of substantiated decision rules and analysis techniques for investing in the hedge situation or other related investments.

In summary, it is obvious that the field of security analysis is very extensive. This thesis deals with only a small segment of this vast area; the stock market hedge situation. For unknown reasons, this segment has largely been ignored by researchers. The areas of fundamental and technical analysis have been relatively unchanged in development since their inception. Currently authors such as Darvas (8) have advocated use of fundamental and technical techniques together. The current state of the art in this area could be called the techno-fundamental approach (8). Work is continuing in the area of the random walk theory, but to date, this work is largely empirical. There are no known consistently successful applications of this theory in existence. Research is also continuing in use of the Markowitz portfolio selection model. Perhaps the greatest advances will ultimately be made in this area. It is not inconceivable that the Markowitz model could be applied to the stock market hedge situation in the future.

Methodological Aspect of Research

As previously stated, this portion of the literature survey deals only with some of the methodology actually used in developing the hedge situation system and its inherent decision rules. Some of the most

important analytical procedures used in developing this thesis deal in the broad area of mathematical probability. Considerable use is made of the computation of expected values. Almost any mathematical probability text explains this concept and how the concept is applied (22). The use of expected values in decision making is included in the decision theory works by Raiffa (25), White (34), and Kassouf (12) to name a few.

Another portion of probability theory that is used extensively is probability assessment. This too plays a major role in this thesis. One of the best references encountered, dealing with probability assessment in general is by Bartos (2). His discussion reviews the techniques for obtaining probability assessments by dividing these techniques into four categories; (a) assumed distributions, (b) ranking techniques, (c) indirect techniques, and (d) direct techniques. The assumed distribution technique starts by assuming a specific type of probability distribution over the events in question. The values of the necessary parameters are applied thus defining a specific distribution. Work in applying this technique has been done by Lamb (14). He used a two stage process for extracting information from a subject and represented it in a probability distribution (2). Ranking techniques involve dividing the range of outcomes into a fixed number of intervals, and then ranking the intervals in ascending order according to the expected relative probabilities of occurrence associated with each (2). Kendall (13) and Smith (29) have developed sophisticated procedures for applying this technique. Inadequacies of ranking techniques are presented by Green (11).

Indirect techniques are those which obtain a subjective probability distribution through the use of analogies, whereas direct

techniques require an explicitly constructed distribution (2). Winkler studied both of these techniques (35). Bartos concluded, based upon Winkler's work, that

> ...The methods outlined by Winkler require no prior assumptions about the shape of the distribution. The direct technique [does] permit the assessor to see how his answers affect his distribution; a situation which is often lacking in other techniques. (2)

The remaining methodology used in the development of this study is basic mathematics coupled with extensive use of graphical representations. This methodology will be self-explanatory as the single decision process and the dynamic multi-decision process are developed in succeeding chapters.

CHAPTER III

SINGLE DECISION PROCESS

General Concept

The basic reason for establishing a stock market hedge situation is to insure profitability over a wide specific price range of the common stock. This, in essence, is a tradeoff an investor makes between profits and risk. Since the hedge situation generally provides profitability over a wider price range, the risk is reduced; but at the same time, profits also are generally reduced.

In developing the solution to the single decision problem, the general concept is to determine the candidates for potential hedge situations, and to determine the terms and conditions of the warrants. Once the potential hedge situations have been selected, the following general procedures will ensue for each candidate. First, a general price range of the common stock at the time the warrant expires will be established. Second, a profit graph will be constructed with return and common stock price at the expiration of the warrant as the two axes. Various profit curves will then be plotted on this graph. A probability distribution will be assigned within the previously established common stock price range. Following these procedures, additional profit curves may be constructed, various tests and possible adjustments made, and determination of expected return will be accomplished. All of these steps will ultimately lead to the priority ranking of the potential hedge situation

candidates. These general procedures will be covered in detail throughout the remainder of this chapter.

The primary emphasis in the development of the general procedures and rules just outlined is toward practicality. That is, these procedures have been established with a view toward minimal complexity, computational ease, readily available information, and relatively small time requirements. An investor inclined toward this type of investment should be able to apply these procedures with a minimum amount of difficulty.

Discussion of the Procedures

An investor considering a hedge situation investment must first determine the possible candidates for this type of investment. As described in Chapter I, the warrant must have an expiration date. This is of prime importance in satisfying the previously stated assumptions upon which this thesis is based. Another criterion for selection of candidates is that the warrants should trade on either the New York Stock Exchange (NYSE) or the American Stock Exchange (ASE). This is not a hard and fast rule; however, generally warrants trading on either of these exchanges can easily be sold short as there are sufficient warrants to be borrowed for short sales. Short sales of warrants listed on regional exchanges or in the over-the-counter market can be accomplished if there are sufficient warrants available for borrowing. Inquiries through brokers would have to be made to ascertain the availability of warrants for short sales if consideration was given to including warrants from these sources. For these reasons, it is recommended that only warrants trading on the NYSE or ASE be included in selecting potential hedge situation candidates.

A list of warrants and their prices trading on the NYSE or ASE can

be compiled from daily newspapers or other financial newspapers such as the Wall Street Journal. Once this list is compiled, the terms and conditions of the warrant can be obtained. The most comprehensive source for obtaining the required information is Moody's Manual available in most libraries and brokerage offices. As previously stated, the basic information that must be obtained for each warrant is (a) the expiration date of the warrant, (b) the conversion price of the warrant into shares of common stock, and (c) the number of shares of common stock into which each warrant can be converted. All of this information is available in Moodys as well as other information such as corporation earnings, common stock price ranges, company products, etc. which will be beneficial for the investor to know. With the above listed required information, and after eliminating all perpetual warrants from consideration, the investor has the necessities for evaluating each of his potential hedge situations.

With the list of situations compiled, the investor must first establish a general price range that the common stock price will be in when the warrant expires. Naturally, the longer the period of time until the warrant expires, the more difficult this will be. The procedures to establish this price range involve several steps. It may seem that some of these steps are rather arbitrary or subjective; however, these steps are generally based upon historical data that is easy to obtain, easy to compute, and most important of all, these steps logically lead to the establishment of a price range that takes into consideration all of the initial information that the investor has.

The first step in establishing the price range is to compute the average of the common stock high prices and low prices for the preceding

five years. The high and low price for each year is listed in Moodys. Five years of historical data were selected because this period of time in the past two decades, has included both an upward and downward stock market move. Here again, the five years of historical data is not a hard and fast rule. If an investor feels more comfortable using a different number of years of historical data, this is his perogative. The average high price is computed by summing the high prices for each year of historical data and dividing this sum by the number of years as shown in Equation (2). The average low price is computed in a similar manner except the low prices for each year are summed and divided by the number of years as shown in Equation (3).

$$\text{Average High Price} = \frac{1}{N} \sum_{i=1}^{N} (\text{Maximum Price})_i \,, \tag{2}$$

$$\text{Average Low Price} = \frac{1}{N} \sum_{i=1}^{N} (\text{Minimum Price})_i \,, \tag{3}$$

where N = Number of years.

A second step in the determination of the price range is to compute the average of each year's High Price/Earnings (PE) ratio and low PE ratio. The per share earnings of the common stock can be obtained from Moodys. Again, five years of historical data are used for the same reasons as before. The average high and low PE ratios are determined from the following equations:

$$\text{Average High PE Ratio} = \frac{1}{N} \sum_{i=1}^{N} \left(\frac{\text{Maximum Price}}{\text{Earnings}} \right)_i \,, \tag{4}$$

$$\text{Average Low PE Ratio} = \frac{1}{N} \sum_{i=1}^{N} \left(\frac{\text{Minimum Price}}{\text{Earnings}} \right)_i , \qquad (5)$$

where N = Number of years.

In the event a company produced a deficit in earnings the PE ratio is considered to be zero.

The Third step in the determination of the common stock price range is to compute each company's average growth rate. The growth rate is the compounded percent change in a company's per share earnings from one year to the next. Once again, five years of historical data are used for this computation. The average growth rate is computed from the following equation:

$$\text{Earnings (Year 1)}(1+\bar{g})^{N-1} = \text{Earnings (Year N)} \quad \text{Where } N = \text{Number of years}, \ \bar{g} = \text{average growth rate} \qquad (6)$$

The next step in the determination of the common stock price range is subjective in nature; yet, very important. This step is to determine the long-term trend of the stock market. The reason that the trend of the market must be determined is because most common stocks move generally along with this trend. If the trend of the market as a whole is up, then an individual common stock will likely also move up. Because of this, it is desirable to "bias" the predicted common stock price range to the upside if the trend of the market is up and to the downside if the market trend is down. This is done in anticipation of the direction the stock will likely move. The trend of the stock market may be determined by considering five factors, each of which requires a subjective decision. The first factor is so called "expert" opinion. Many writers

in financial publications have considerable experience and expertise in matters dealing with the economy in general and the stock market in particular. Many of these writers express their views as to future trends and actions in the market. While the stock market is generally considered to be quite unpredictable, these writers attempt to forecast upcoming trends and developments based upon their knowledge, experience, and information. Most leading financial publications such as Wall Street Journal, Barrons, Forbes, Financial World, etc., have regular columnists who periodically express their opinions. By regularly reading a few selected columns, an investor can determine what the general consensus of opinion of the "experts" is concerning future stock market trends.

The second factor to assist in determining the stock market trend is data concerning the general economic conditions. Specifically, the direction of (a) the country's gross national product, (b) employment figures, (c) disposable personal incomes, (d) index of industrial production, and (e) the cost of living index (26). Financial publications such as Financial World publish these statistics on a weekly basis. Again a subjective decision for this factor is required; however, the trend of each of the five items just listed can be determined.

The third factor to assist in determining the trend of the market is the data concerning more specific economic conditions (26). Data concerning the trend of (a) automobile and building production, (b) department store sales, (c) wholesale price levels, (d) dividends and earnings of companies, and (e) the federal reserve policy (the prime interest rate, reserve requirements, credit restrictions, etc.), are also published in leading financial publications periodically. The trend of each of these five items can also be determined.

The fourth factor to consider is a group of indices and data published pertaining to the stock market itself. The Wall Street Journal and Barrons, to name two, publish information concerning (a) composite averages (such as Dow Jones Industrial Average, NYSE index, etc.), (b) ratio of advances to declines, (c) new highs and new lows, (d) volume of trading, (e) short interest, and (f) odd lot trading. Once again, these data can be examined and a subjective decision made as to the trends of each of the above six items.

The final factor to consider in determining the trend of the stock market are the technical considerations through the use of vertical line charts and figure charts. It is beyond the scope of this thesis to explain how to apply technical techniques to the various charts. If an investor is familiar with the technical aspect of the stock market he can make a subjective decision for this factor. If the technical approach is not familiar to an investor he can alternately secure the opinions of the technical analysts from financial publications such as Forbes which prints a regular column devoted to this approach.

When considering each of the five factors just discussed, the investor should assign a value of +1 to each factor he subjectively determines indicates an upward trend. A value of -1 should be assigned to any factor he determines indicates a downward trend. To determine the trend of the stock market, the values for all five factors are added. If the sum is positive, the trend is up. If the sum is negative, the trend is down.

With the foregoing steps completed the common stock price range

can now be determined. If the trend of the stock market is up, the minimum price of the price range is the greater of the average low price computed from Equation (3) or the average low PE ratio, computed from Equation (5), multiplied by the projected earnings.

$$\text{Projected Earnings in Period N} = \text{Current Earnings} \times (1 + \text{Average Growth Rate})^{N} \quad (7)$$

where N = Number of periods

The maximum price is similarly determined by selecting the greater of the average high price or the average high PE ratio multiplied by the projected earnings. This tends to "bias" the price range to the upside.

If the trend of the stock market is down, the minimum price is the smaller of the average low price or the average low PE ratio multiplied by the projected earnings. The maximum price is the smaller of the average high price or the average high PE ratio multiplied by the projected earnings. This procedure tends to "bias" the price range to the downside. To facilitate future computations, in all cases round maximum price range values to the next higher whole price if a fractional portion exists, and round minimum price range values to the lowest whole price by truncating any fractional portion.

At this point a general price range has been established for the initial evaluation. The current price of the common stock should lie within this range. It is necessary in order to accomplish the remaining evaluations that the current common stock price lie within the price range. In the event the current common stock price is outside the projected price range, this price range must be adjusted. It is

important to realize the significance of the price range since once it is established it is a guide from which many of the future evaluations are made. Under ideal conditions the price of the common stock would remain within the price range; however, these conditions may not always prevail. Consequently, provisions are incorporated initially, in evaluating potential hedge situations, and in the multi-decision process to modify the price range as appropriate. If the initial current common stock price lies outside of the computed price range, or if at any time during the single decision or dynamic multi-decision processes the common stock price moves outside the determined price range, the price range is modified following these procedures:

a. The standard deviation is determined from the following equation:

$$\text{Standard Deviation} = \left[\frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{N - 1} \right]^{1/2} , \qquad (8)$$

where: N = Number of prices

$\bar{x}$ = Average price

x_i = price$_i$

b. Determine the average common stock price, $\bar{x}$, from previous historical data concerning common stock prices using this equation:

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{N} \text{Price}_i \ , \quad \text{where} \quad N = \text{Number of prices in previous historical data}$$

c. Solve for the standard deviation using Equation (8). As a guide, a minimum of one month of previous stock prices should be used to insure an adequate sample is taken. It is possible to use more historical data, as desired, to compute the standard deviation.

d. Given the common stock price that has exceeded a price range limit, add three standard deviations to this price and round off as previously described, thus establishing a new upper price range limit. Similarly, subtract three standard deviations from this given common stock price and round off as previously described, thus establishing a new lower price range limit.

Probably the most important and most difficult procedure is the assignment of a probability distribution to the price range of the common stock. Recall from Chapter II that probability assessment techniques are generally categorized as (a) assumed distributions, (b) ranking techniques, (c) indirect techniques, and (d) direct techniques. When dealing with the stock market and individual stocks it is difficult at best to forecast how the stock prices will move in the future. It is possible to assume a known probability function over the price range but no known researchers have effectively and consistently done this to date. As a result, since this thesis is oriented toward a practical application, a subjective discrete probability distribution, in histogram form, will be assigned over the computed price range of the common stock. Even this is extremely difficult to do, particularly for someone not experienced in stock market operations.

The best approach to assigning probabilities is to divide the price range into intervals. The number of intervals should not exceed

six as it is very difficult to subjectively assign probabilities to any more than this number of intervals. Research by Moore and Baker (24) related to the area of subjective assessment substantiates this. When the intervals are established, the investor should attempt to rank these intervals in the order he feels they will likely occur. Procedures similar to those described by Raiffa (25) can be used to assist in the ranking of these intervals, as well as the subjective assignment of probability to each of the intervals. The investor has knowledge concerning the trend of the stock market and it is likely each stock will generally follow this trend. He can determine the current price to earnings ratio and multiply this PE ratio by the projected earnings to get some idea of where the price might be in the future. From immediate historical prices (one week to one month old) he can determine what the immediate trend of the stock prices are. Using this available information, the investor can complete this ranking. Following this, each interval should be assigned a probability with values in descending order so that the sum of all probabilities is equal to one. This is a time consuming process. When this has been accomplished, the probability distribution can be plotted in histogram form for later use. To determine the probability for each price the probability for each interval is divided by the number of prices in the interval.

Oftentimes the investor may feel he has no idea what the distribution of future stock prices should be. In this case he should assume that each interval has an equal probability of occurrence. If the investor feels he cannot effectively construct a probability distribution, yet he feels the stock will move in one certain direction, then

he can assign probabilities such that those intervals in the direction of movement will receive greater values.

Unfortunately this method of probability assessment is rather unscientific. It must be remembered however, that the primary advantage of the hedge situation is that it precludes loss on an investment so long as the common stock price remains in the selected range. Even though the assessed probability distribution is erroneous, a loss will not result as long as the common stock price remains in the price range.

The concept of the adjusted conversion price of a warrant is used extensively in the single decision process (33). It is known that a warrant plus the appropriate amount of money may be converted into a specific number of shares of common stock. If a company pays a dividend in stock or splits its common stock there are provisions which also change the terms of the warrant to protect its value. Usually the number of shares of common stock into which the warrant may be converted is modified to reflect the change and prevent dissolution of the warrant. It is not uncommon for a warrant to be converted into more than or less than one common stock share. To compensate for this in evaluating all potential situations, each conversion price is divided by the number of common shares into which each warrant may be converted. This value is called the adjusted conversion price (ACP) and is the value at which maximum profit in a hedge situation will occur when the number of short warrants is greater than or equal to the number of shares of long common stock.

$$\text{Adjusted Conversion Price (ACP)} = \frac{\text{Conversion Price}}{\text{Number of shares of Common Stock into which each Warrant can be converted}} \qquad (9)$$

Another tool extensively used throughout this thesis is the profit graph. In this graph the vertical axis is labeled percent profit or return, and the horizontal axis is labeled common stock price at expiration of warrant. The various linear profit curves are plotted on this graph. The short warrant only profit curve is plotted by drawing a straight line from the profit axis along the 100% line, or 1.0 line if decimal notation is used, to the adjusted conversion price. A second line is drawn from the point (1.0, ACP) through the short only intercept on the common price axis. This intercept is determined by equating:

$$\begin{array}{l}\text{Theoretical Value of} \\ \text{Warrant at Expiration}\end{array} = \begin{array}{l}\text{Short Sale Price} \\ \text{of Warrant}\end{array}$$

Using Equation (1) this equation becomes:

$$\begin{array}{l}\text{Short Only} \\ \text{Intercept on} \\ \text{Common Price} \\ \text{Axis}\end{array} \times \begin{array}{l}\text{No. of Common} \\ \text{Shares into} \\ \text{Which Warrant} \\ \text{Can Convert}\end{array} - \begin{array}{l}\text{Conversion} \\ \text{Price}\end{array} = \begin{array}{l}\text{Short Sale} \\ \text{Price} \\ \text{of} \\ \text{Warrant}\end{array}$$

This equation can be solved resulting in the following:

$$\begin{array}{l}\text{Short Only} \\ \text{Intercept on} \\ \text{Common Price} \\ \text{Axis}\end{array} = \frac{\begin{array}{l}\text{Short Sale Price} \\ \text{of Warrant}\end{array} + \begin{array}{l}\text{Conversion} \\ \text{Price}\end{array}}{\begin{array}{l}\text{Number of Common Stock Shares} \\ \text{into which Warrant can Convert}\end{array}} \qquad (10)$$

The long only profit curve is plotted by drawing a straight line through the current common price value on the common price axis to the value -100% on the profit axis.

The intersection point of the long only curve and the short only

curve may be determined by writing the equations of the profit curves and solving. The equation for the short only curve in the form $y = mx + b$ is as follows:

$$y = \text{Slope}(x) + (1 - \text{Slope}(\text{ACP})), \text{ where Slope} = \frac{-1}{\begin{matrix}\text{Short only}\\ \text{Intercept}\\ \text{on Common}\\ \text{Price Axis}\end{matrix} - \text{ACP}} \qquad (11)$$

The equation for the long only profit curve in the form $y = mx + b$ is as follows:

$$y = \frac{1}{\begin{matrix}\text{Current Price of}\\ \text{Common Stock}\end{matrix}} x - 1 \qquad (12)$$

Other profit curves may be constructed by determining return at three points along the common stock price axis. One of these points must be the adjusted conversion price, one point must be to the left of the ACP, and one point must be to the right of the ACP. The procedures necessary to obtain the value of return at any particular common stock price are as follows:

a. Write the particular common stock price for which the return is to be calculated.

b. Determine the theoretical warrant value at this point using Equation (1).

c. Determine the gain on the warrant:

$$\begin{matrix}\text{Gain on}\\ \text{Warrant}\end{matrix} = \begin{matrix}\text{Current Warrant}\\ \text{Price}\end{matrix} - \begin{matrix}\text{Theoretical}\\ \text{Warrant Value}\end{matrix} \qquad (13)$$

d. Determine the gain on the common stock:

$$\text{Gain on Common} = \text{Particular Common Stock Price} - \text{Current Common Stock Price} \quad (14)$$

e. Determine the amount of money (M) to establish desired ratio:

$$M = \text{Current Warrant Price} \times \text{Number of Warrants} + \text{Current Common Price} \times \text{Number Shares of Common} \quad (15)$$

f. Determine the return at the particular common price:

$$\text{Return} = \frac{\text{Gain on Warrant} \times \text{Number of Warrants} + \text{Gain on Common} \times \text{Number Shares of Common}}{M} \quad (16)$$

When the return for each of the three required particular common stock prices have been calculated, using the above procedures, the linear profit curves can be constructed by drawing two straight lines. One line is from the return point determined at the ACP through the return point calculated at the common price to the left of the ACP. The other line is from the return point calculated at the ACP through the return point calculated at the common price to the right of the ACP.

Another procedure, concerning the profit curves, that is used extensively throughout the evaluation is the determination of the ratio of short warrants to long common stock profit curves that pass through each of the price range points on the common price axis of the profit graph. To determine the hedge ratio that has a profit curve passing through the lower price limit the following are equated:

Gain on Warrants if Common Stock Price is at the Lower Range Point X Ratio = Loss on the Common Stock at the Lower Range Point

This can be expressed as:

$$\left[\text{Short Sale Price of Warrant} - \left(\text{Lower Range Point} \times \text{No. of Common Shares into Which warrant can convert} - \text{Exercise Price}\right)\right] \text{Ratio} = \quad (17)$$

$$\text{Purchase Price of Common Stock} - \text{Lower Range Point}$$

This equation can be solved for the ratio. If the ratio is less than one, more long common than short warrants will be established. If the ratio is greater than one, more short warrants than long common stock will be established. It is important to note in Equation (17) and in subsequent equations dealing with theoretical warrant value that if the theoretical warrant price computes to a value less than zero, it must be considered zero since warrant prices will not assume a negative value in trading on the various exchanges. Refer to Equation (1) to review procedures for determining theoretical warrant values.

To determine the hedge ratio that has a profit curve passing through the upper price limit the following are equated:

Loss on Warrants if Common Stock Price is at the Upper Range Point X Ratio = Gain on the Common Stock at the Upper Range Point

This can be expressed as:

$$\left[\left(\begin{array}{c}\text{Upper}\\ \text{Range}\\ \text{Point}\end{array} \times \begin{array}{c}\text{No. of Common}\\ \text{Shares into}\\ \text{Which Warrant}\\ \text{Can Convert}\end{array} - \begin{array}{c}\text{Exercise}\\ \text{Price}\end{array}\right) - \begin{array}{c}\text{Short Sale}\\ \text{Price of}\\ \text{Warrant}\end{array}\right] \text{Ratio} = \qquad (18)$$

$$\text{Upper Range Point} - \text{Purchase Price of Common Stock}$$

Once again, this equation can be solved for the ratio.

When the short warrant/long common stock ratios are determined from Equations (17) and (18) the profit curves can be plotted following the procedures previously outlined. When these curves are plotted, a problem may exist. It is possible that one or both of these profit curves do not span the entire price range. This means that there are points along the common price axis, within the determined price range, that will result in a loss on the investment if the common price reaches these points when the warrant expires. There is no way to change any ratios to rectify this difficulty. The hedge investor has two alternatives when this situation occurs. First, he may reject the potential situation and eliminate it from consideration since it will not insure against loss as long as the common stock price remains within the price range. His second alternative is to revise his price range since he cannot alter the ratio. He may arbitrarily move one, or both, of his price range limits to a point where one or both of the profit curves cross the common price axis thus narrowing the range. If he elects to narrow the price range the hedge candidate can continue to be considered; however, if the narrowed price range is exceeded a loss could conceivably occur on the investment.

Another procedure used in evaluating each of the potential hedge

situation candidates is the determination of expected return. It is known that:

$$E(\text{return}) = \int_A^B g(x)\,f(x)\,dx, \quad \text{where: } \begin{array}{l} g(x) = \text{probability distribution} \\ f(x) = \text{profit curve} \\ A \text{ to } B = \text{Common price range} \end{array}$$

However, as previously discussed, a subjective discrete probability distribution over the common stock price range has been selected for use in this thesis, consequently:

$$E(\text{return}) = \sum_{i=A}^{B} \begin{array}{c}\text{Return at} \\ \text{Price } i\end{array} \times \begin{array}{c}\text{Probability} \\ \text{of Price } i\end{array} \qquad (19)$$

The equation for determining return at any given point was presented in Equation (16).

Since the probability distribution has been established, and since the return for each price within the price range can be determined, the expected return can easily be determined. The profit curve that results in the greatest expected return is selected for further evaluation.

A tool used in the evaluation of the potential hedge candidates is to take into consideration the time value of money through use of discounting and determination of present value.

$$\text{Present Value} = \sum_{i=0}^{N} \text{Return}_i (1+r)^{-N}, \quad \text{where } \begin{array}{l} r = \text{rate of interest} \\ N = \text{number of periods} \end{array}$$

The return is computed as previously discussed. This return will be forthcoming when the warrant expires. As a result, the above equation can be rewritten as

$$\text{Present Value} = E(\text{return})(1+r)^{-N} \qquad (21)$$

The rate of interest to be used should be the rate at which money can be borrowed. The most practical source of borrowing for securities accounts is through the use of margin accounts. The current margin rate of interest should be the rate used in this computation. Interest is paid monthly on margin accounts; therefore, the value for N should be the number of months until the warrant expires.

This concludes the explanation of procedures that are used in the single decision process. The next section of this chapter will illustrate the sequence in which these procedures are applied.

Application of Procedures and Rules

This section will fully explain the process that each of the potential stock market hedge situation candidates will go through, ultimately leading to the priority ranking of these situations. References, by page number, will be made to the preceding section to assist in the understanding of this process.

Figure 2 is a diagram which will further assist in the understanding of the entire process and the developed decision rules.

The following procedures must be accomplished, in order, for each potential candidate:

a. Compute the average high and low price (Page 25).

b. Compute the average high and low PE ratio (Page 25).

c. Compute the average growth rate (Page 26).

d. Determine the trend of the stock market (Page 26).

If the trend of the stock market is up, follow Rule 1.

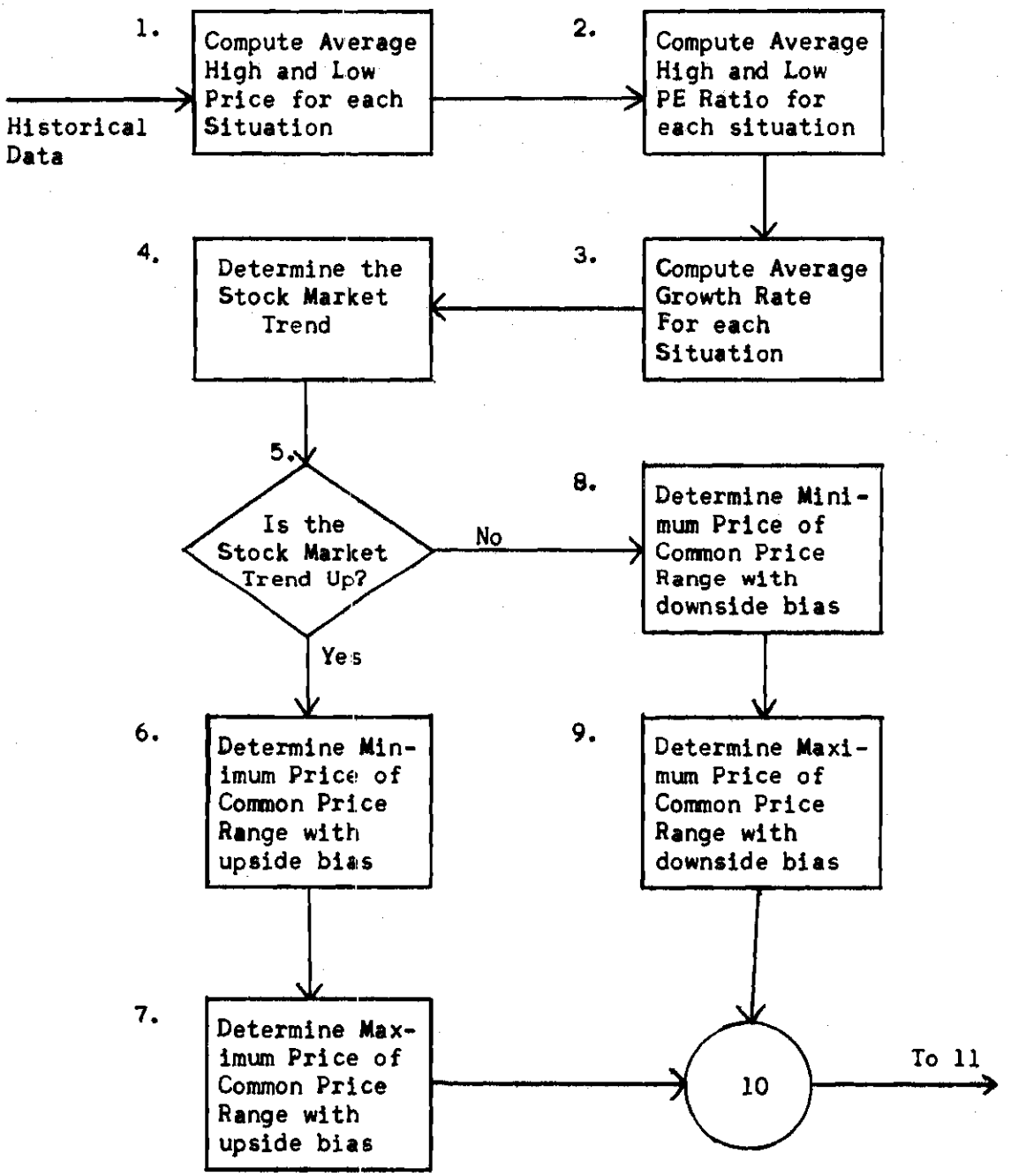

Figure 2. Flow Diagram of Single Decision Process.

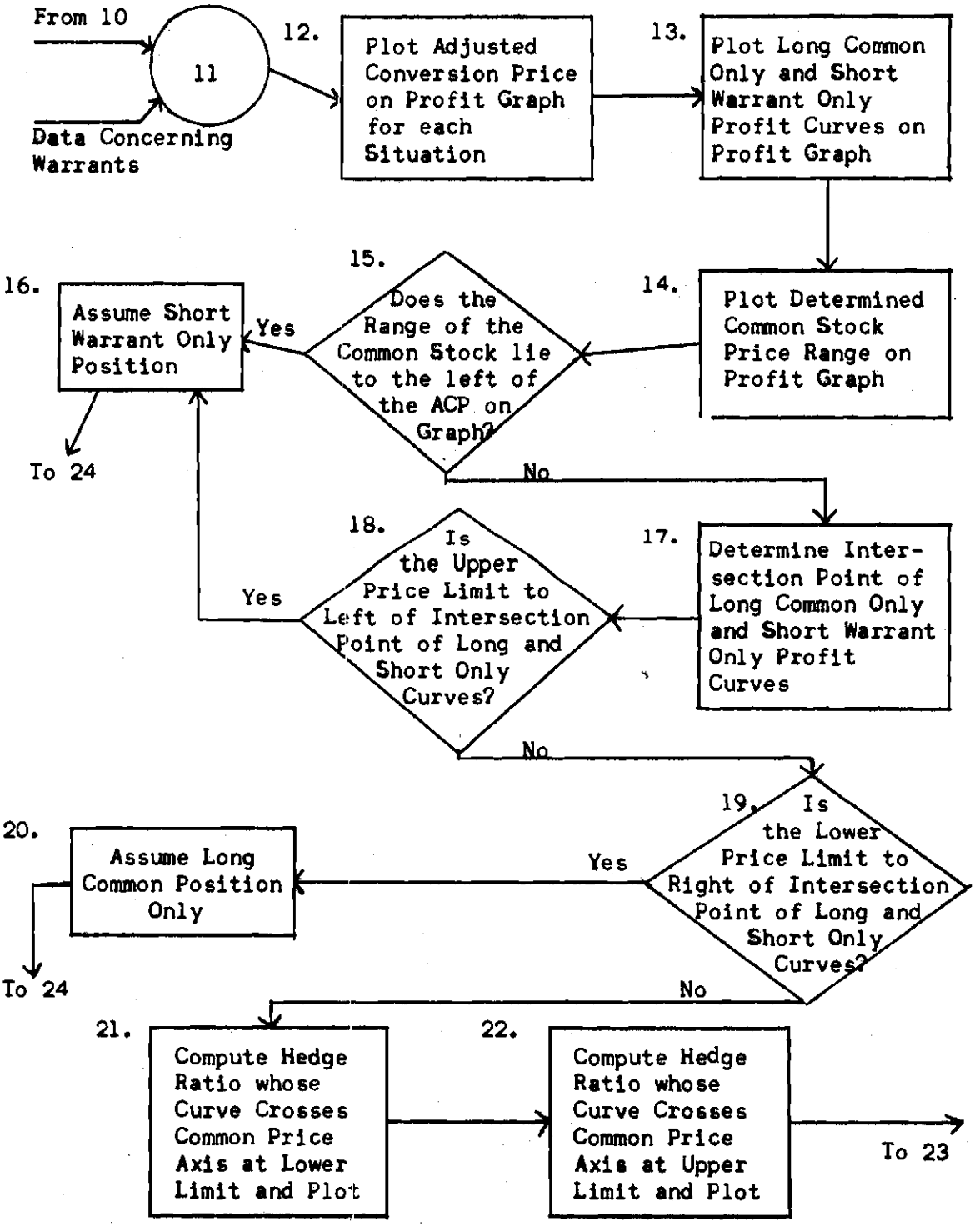

Figure 2. (Continued)

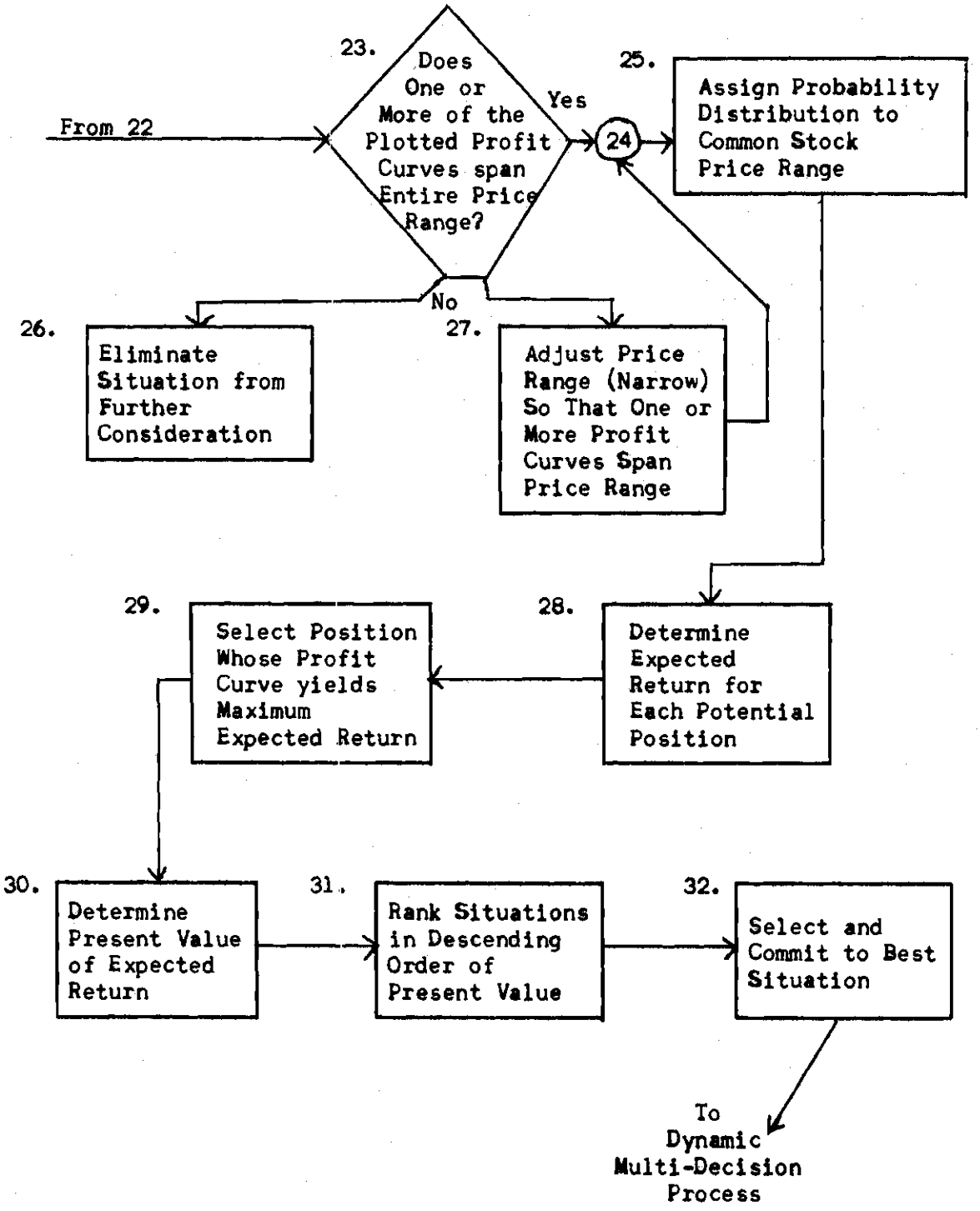

Figure 2. (Continued)

If the trend of the stock market is down, follow Rule 2.

Rule 1: Determine the minimum and maximum price of the common stock price range with an upside bias (Page 29).

Rule 2: Determine the minimum and maximum price of the common stock price range with a downside bias (Page 29).

When the price range is established, the following procedures must be accomplished:

a. Determine the adjusted conversion price and plot this point on the profit graph (Page 33).

b. Plot the long common stock only profit curve on the profit graph (Page 34).

c. Plot the short warrant only profit curve on the profit graph (Page 34).

d. Plot vertical lines through the two price range points along the common price axis of the profit graph.

If the entire price range of the common stock lies to the left of the adjusted conversion price follow Rule 3.

Rule 3: Assume a short warrant position only.
The reason for this is because maximum profit will always occur by establishing a short warrant position when the above condition exists. See Figure 1 for illustration.

Next, determine the intersection point of the long common only and short warrant only profit curves (Page 35).

If the upper price range limit is to the left of the intersection point of the long only and short only profit curves, follow Rule 3.

If the lower price range limit is to the right of the inter-

section of the long only and short only profit curves, follow Rule 4.

Rule 4: Assume a long common position only.

The reason for this is because maximum profit will always occur along the long only profit curve when the above condition exists. Again see Figure 1 for illustration.

If no recommendations have been made for establishing positions, the following procedures must subsequently be accomplished:

a. Determine the hedge ratio whose profit curve crosses the common price axis at the lower price range point (Page 37).

b. Plot this profit curve on the profit graph (Page 36).

c. Determine the hedge ratio whose profit curve crosses the common price axis at the upper price range point (Page 38).

d. Plot this profit curve on the profit graph (Page 36).

If the conversion terms of the warrant specify that the warrant is converted into one share of common stock then Rule 5 should be followed.

Rule 5: Plot the profit curve for a ratio of 1:1 on the graph.

This rule is necessary when the above condition exists because the profit curve for a 1:1 hedge ratio reaches its maximum at the adjusted conversion price and profit continues at that same value of return regardless of how far up in price the common stock moves. The ratio of 1:1 might not be included in determining the previously listed procedures but it definitely must be considered.

Following the plotting of the appropriate profit curves, one last condition must be checked. If none of the plotted profit curves span the entire price range of the common stock then Rule 6 or Rule 7 as desired applies. This means that there are prices within the price

range where a loss is possible.

Rule 6: Eliminate the potential candidate from future consideration.

Rule 7: Adjust the price range (narrow so that one or more of the plotted price curves cover the entire range.

If Rule 6 is followed, nothing else remains for the potential candidate. If Rule 7 is followed or if the original plotted profit curves span the price range of the common stock then the procedures for assigning probabilities to prices within the common stock price range must be accomplished (Page 31).

Once the probabilities have been assigned, the following procedures should be accomplished.

a. Determine the expected return for each potential position (Page 39).

b. Select the hedge ratio whose profit curve yields maximum expected return.

c. Determine the present value of the expected return (Page 39).

When all potential hedge situation candidates have been evaluated by applying the rules and procedures, presented, these candidates should be ranked in order of highest present value to lowest present value.

The investor is now in a position to choose from his list of possible hedge situations. It is beyond the scope of this thesis to cover detailed procedures for allocating funds among these possible situations. This type of analysis is suggested as a possible future extension of this study.

At this point the rules and procedures for the single decision process have been fully developed. It will be assumed that an investor has chosen one of these situations as his investment vehicle. He is now ready to consider the dynamic forces that will be applied to his selected hedge situation, and to act on these forces to insure the profitability of his investment.

CHAPTER IV

DYNAMIC MULTI-DECISION PROCESS

General Concept

The dynamic multi-decision process is a general process or system that controls one or more of the stock market hedge situations that are selected for investment from the single decision process. Most of the basic procedures are similar to the procedures of the single decision process. Once again, lack of complexity and computational ease have been the primary considerations in developing this phase of the process.

In general, each hedge situation is periodically reviewed, based upon new available information concerning the trend of the stock market and price action of the common stock, to ascertain if the price of the common stock is remaining within the determined price range for that stock. As long as the price remains in this price range the investor is assured of sustaining no loss on his investment. If the trend of the stock market changes, or if the price range of the common stock is exceeded, additional checks, computations, and adjustments are made which ultimately will place the investor back in such a position so as to insure profitability on his investment.

One basic assumption in this dynamic process is that should an occasion arise where an investor would have to revise his position, he does have the available financial resources to accomplish this

change. The remainder of this chapter will include a detailed explanation of the procedures and rules to be followed in this process.

Discussion of the Procedures

All of the procedures used in the development of the dynamic multi-decision process are the same as the procedures used in the single decision process except in the determination of the return when plotting new profit curves and computing expected return. In addition, the order of these procedures has been changed somewhat.

When an investor has decided upon the hedge situation he wishes to invest in, he commits to this situation using the appropriate short, long, or hedge position. As time passes it is unlikely that the prices of the common stock and the warrant will be exactly the same, on the same day, as when he made his first commitment. For this reason the investor must be prepared, if necessary, to revise his commitment under future new prices. He must however, consider his previous commitment and its potential for loss or gain.

When a price range limit is exceeded a loss is usually probable on the investment. For example, if an investor was committed to an investment in a hedge ratio whose profit curve crossed from the upper to the lower side of the common price axis at the upper common stock price limit, and if the common stock price exceeded this price limit at the expiration date of the warrant, a loss would result. Consequently, the investor should revise his price range, following the procedures presented in Chapter III, and attempt to revise his previous position to preclude any loss. It is important to realize that the investor may

have to realize a temporary loss in order to revise this previous position to insure future profitability. The procedures for determining the hedge ratios whose profit curves pass through the revised common stock price limits are identical to the procedures culminating in Equations (17) and (18) in Chapter III.

The only change in the procedures for the dynamic process occurs in the determination of return for constructing the various profit curves and for determining expected return. Referring to Equation (16):

$$\text{Return} = \frac{\text{Gain on Warrant} \times \text{Number of Warrants} + \text{Gain on Common} \times \text{Number Shares of Common}}{M}$$

where the amount of money (M) is expressed as:

$$M = \text{Current Warrant Price} \times \text{Number of Warrants} + \text{Current Common Price} \times \text{Number Shares of Common}$$

Since the previous position should be closed out before the new position is established, a loss or a gain will be realized on the previous commitment. The loss or gain must be considered when making the new commitment. This gain or loss may be determined from the following equation:

$$\text{Gain on Position} = \left[\text{Current Price of Common} - \text{Previous Price of Common}\right] \text{Number Shares of Common} + \left[\text{Previous Warrant Price} - \text{Current Warrant Price}\right] \text{Number of Warrants} \qquad (22)$$

If when solving this equation a positive value is obtained, a gain results on the previous commitment; if a negative value is obtained, a loss results on the previous commitment. This gain or loss must be considered in determining the amount of money (M) required to establish the new position. Therefore, the value of M, or in this case M', is expressed as:

$$M' = \begin{matrix}\text{Current}\\ \text{Warrant}\\ \text{Price}\end{matrix} \times \begin{matrix}\text{Number}\\ \text{of}\\ \text{Warrants}\end{matrix} + \begin{matrix}\text{Current}\\ \text{Common}\\ \text{Price}\end{matrix} \times \begin{matrix}\text{Number}\\ \text{Shares of}\\ \text{Common}\end{matrix} + \begin{matrix}\text{Realized}\\ \text{Gain (-)}\\ \text{Loss (+)}\end{matrix} \qquad (23)$$

The return at any particular common stock price can then be determined, following the procedures outlined in Chapter III, using the value of M' as determined in Equation (23). This will allow the new profit curves to be completed, and will allow the determination of the expected return.

Armed with the procedures presented in detail in Chapter III and the procedures presented here, the investor is prepared to apply the dynamic multi-decision process.

Application of Procedures and Rules

This section will be presented in the same format as the corresponding section in the previous chapter. References, by page number, will again be made to the preceding sections to assist in the thorough understanding of this process.

Figure 3 is a flow diagram which completely traces the dynamic process through. It is designed to assist the investor in his understanding of the developed processes and rules.

Since a commitment has already been made to a hedge situation,

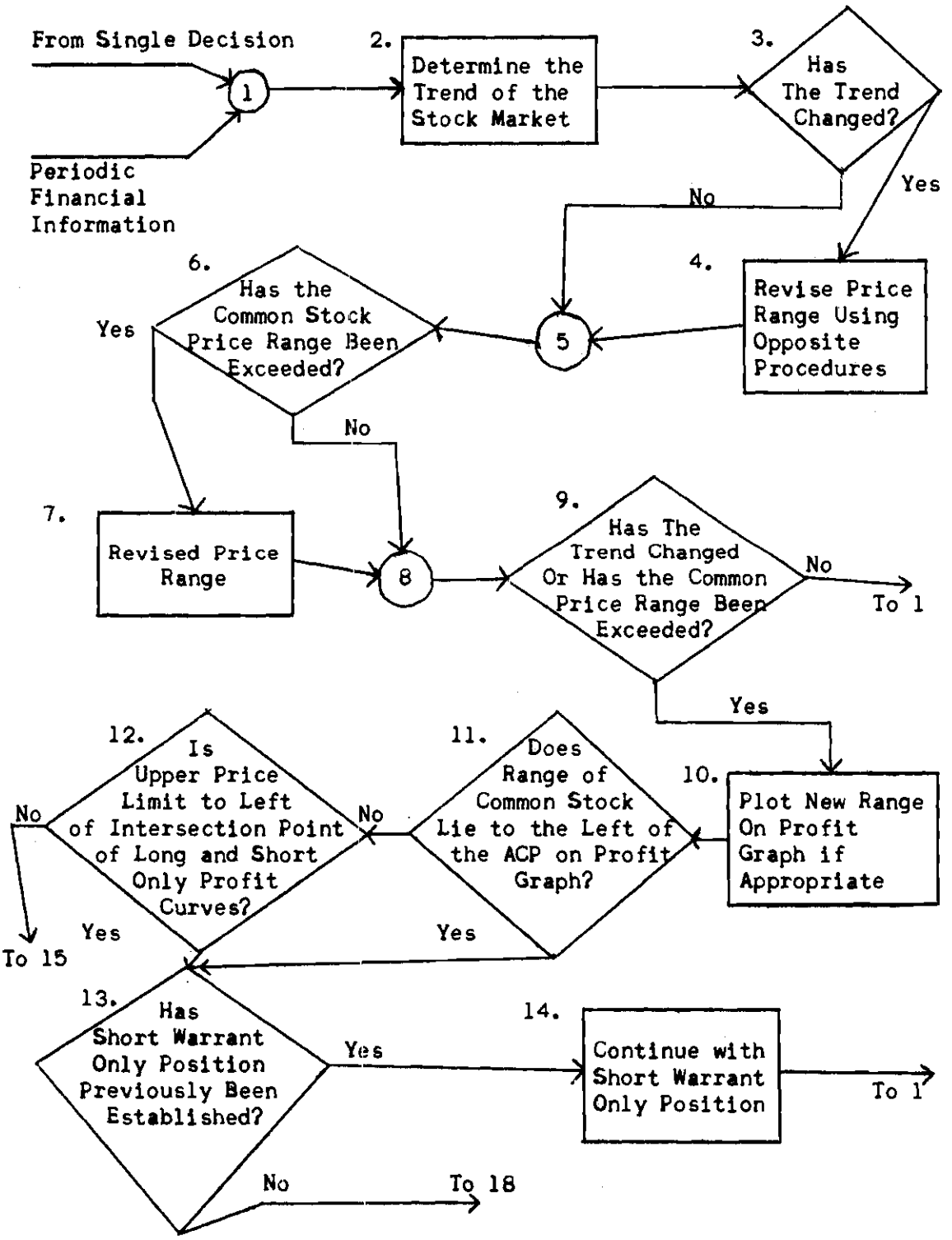

Figure 3. Flow Diagram for Dynamic Multi-Decision Process

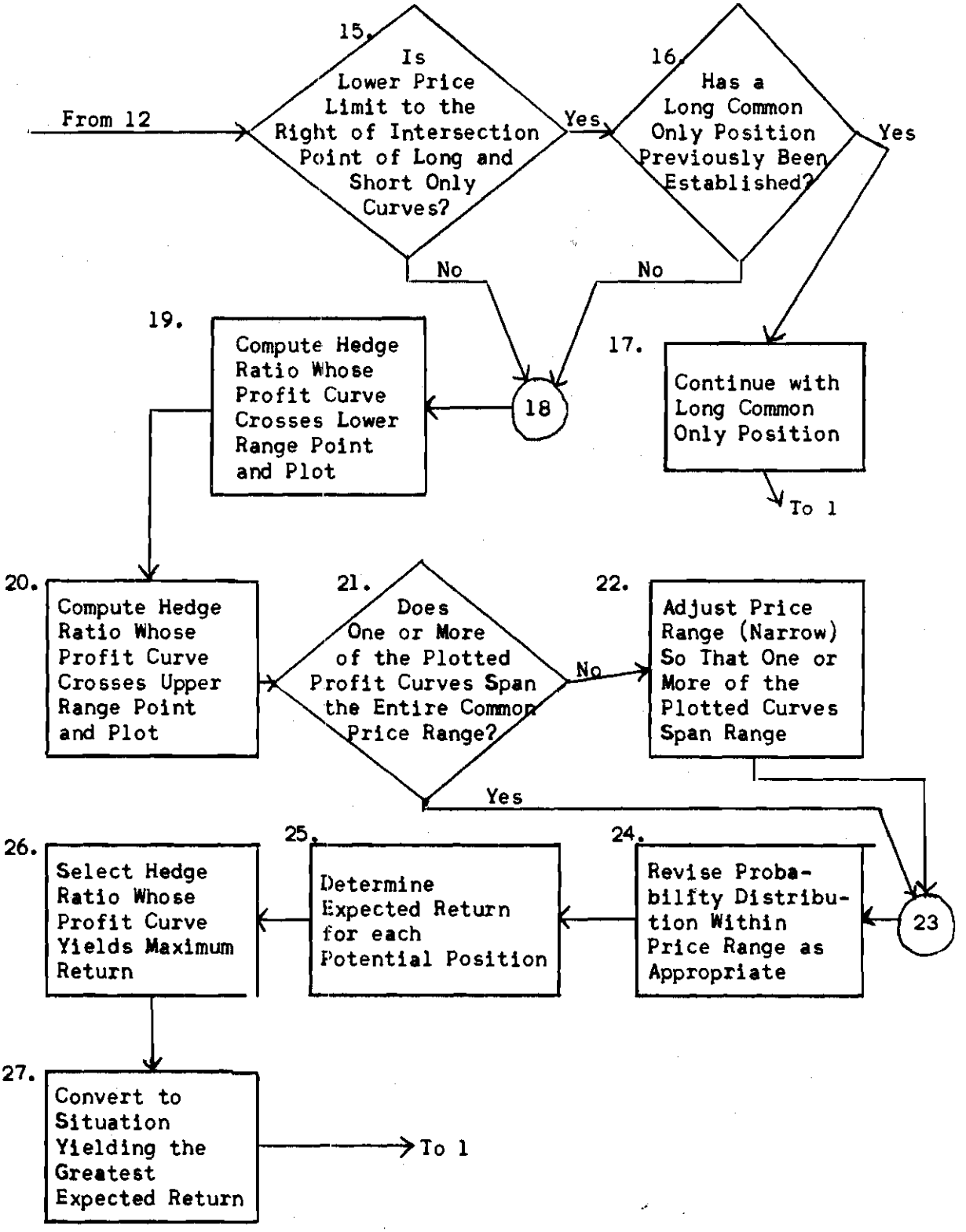

Figure 3. (Continued)

the investor should retain the computations and profit graph developed in the single decision process as this information will be useful for future computations. As time passes, new information will become available to the investor which will influence his future decisions. The most important information will be information, as published in various financial publications, that enables him to update his determination of the trend of the stock market, and that describes the price action of the common stock. Periodically, as desired by the investor, he must determine the trend of the stock market (Page 25).

If the trend of the stock market has changed from the last assessment then Rule 1 should be followed.

Rule 1: Revise price range of common stock following the opposite procedures as used before (Page 29).

Next, a check of current common stock prices must be made to determine if the price range of the common stock has been exceeded.

If the price range of the common stock has been exceeded, follow Rule 2.

Rule 2: Revise the price range of the common stock (Page 30).

If the trend of the market has not changed nor has the common stock price range been exceeded no further steps need be taken until the next periodic check.

If the trend has changed or the common stock price range has been exceeded then Rule 3 is followed.

Rule 3: Plot the new price range on the profit graph.

If the range of the common stock price lies to the left of the adjusted conversion price, and if a short warrant only position has

been established previously then Rule 4 is followed.

Rule 4: Continue with short warrant only position.
This is because maximum profit will continue along the short only profit curve within this price range.

If the range of the common stock price does not lie to the left of the ACP, if the upper common stock price range is to the left of the intersection point of the long common only and short warrant only profit curves, and if a short warrant only position has been established previously then Rule 4 is also followed.

If the range of the common stock price does not lie to the left of the ACP, the upper range is not to the left of the intersection point of the long only and short only profit curves, but the lower price range is to the right of this intersection point, and previously a long common only position was established, then Rule 5 is followed.

Rule 5: Continue with long common only position
This is because maximum profit will continue to be obtained along the long common only profit curve within the price range.

For any conditions pertaining to the location of the price range or previously established positions not specifically stated previously, the following procedures will be accomplished:

a. Compute the hedge ratio that results in a profit curve that crosses through the lower price range point (Page 37).

b. Plot this curve on the profit graph (Pages 36 and 51).

c. Compute the hedge ratio that results in a profit curve that crosses through the upper price range point on the common price axis (Page 38).

d. Plot this curve on the profit graph (Pages 36 and 51).

If none of the plotted profit curves span the entire price range then follow Rule 6.

Rule 6: Adjust (narrow) the price range so that one or more of the profit curves span the price range (Page 38). Alternatively, the position could be closed out since profitability can no longer be assured unless the price range is narrowed. This action could result in a loss on the investment.

The following procedures should be accomplished next:

a. Revise the probability distribution over the price range as appropriate (Page 32).

b. Determine the expected return (Page 39).

c. Select the hedge ratio that indicates the maximum expected return.

d. Close out the present position and establish the new position.

The investor should continually go through this process until the warrant is one or two days from expiration. At this time, the position should be closed. It is felt that this dynamic process will enable the investor to obtain greater profits then if he were to just commit to a hedge situation and make no changes until the warrant expires.

CHAPTER V

ILLUSTRATIVE EXAMPLES

General

This chapter will be devoted to applying actual historical stock market situations to the single decision and dynamic multi-decision processes developed in the preceding chapters. The purpose of these illustrative examples is to demonstrate the potential for investment gains, and the computational procedures involved in leading to various decisions.

Attempts have been made to eliminate any bias from previous knowledge of historical actions in the stock market. The initial data of beginning the single decision evaluation process was randomly selected as February 12, 1968. Another restriction placed upon potential hedge situation candidates was that the various warrants must have expired by the end of 1971 so that the final results could be obtained. All data concerning warrant and common stock prices was obtained from Barrons. An initial list of warrants trading on the American Stock Exchange was compiled. On February 12, 1968 no warrants were trading on the New York Stock Exchange. The expiration date was determined for each warrant. All perpetual warrants and all warrants with expiration dates after December 31, 1971 were eliminated as possible candidates. The potential hedge situation candidates remaining after this screening were: First National Realty and Construction Company, Martin Marietta

Corporation, Pacific Petroleum Corporation, and United Industrial Corporation. None of these corporations was familiar to the author nor was he knowledgeable as to the previous price action of the warrants or common stock of these corporations.

Single Decision Process

The following table lists the pertinent required information for each potential candidate as extracted from the 1968 edition of Moodys Manual (28).

Table 1. Required Information for Hedge Situation Candidates

Corporation and Terms of Warrant	Year	Price Range High	Price Range Low	PE Ratio High	PE Ratio Low	Earnings
First Nat'l Realty:	1967	2.75	.75	0	0	D.55
	1966	1.875	.625	0	0	D1.11
Expires 12/31/71	1965	2.625	1.125	0	0	D.04
Conversion Price: 6.75	1964	3.375	2.0	15.34	9.09	.22
Converts to 1.15 Shares	1963	3.875	2.875	12.92	9.58	.30
Martin Marietta:	1967	26.125	19.125	15.93	11.66	1.64
	1966	27.75	16.625	14.45	8.66	1.92
Expires 11/1/68	1965	25.0	17.75	16.13	11.45	1.55
Conversion Price: 45	1964	20.5	17.125	12.28	10.25	1.67
Converts to 2.73 Shares	1963	22.625	18.0	14.69	11.69	1.54
Pacific Petroleum:	1967	19.5	10.0	36.79	18.87	.53
	1966	14.625	8.625	44.32	26.14	.33
Expires 3/31/68	1965	11.625	8.875	44.71	34.13	.26
Conversion Price: 19	1964	14.125	10.125	67.26	48.21	.21
Converts to 1.1 Shares	1963	14.5	10.25	80.56	56.94	.18
United Industrial:	1967	20.75	13.0	31.44	19.70	.66
	1966	27.75	11.5	26.94	11.17	1.03
Expires 11/15/69	1965	19.5	8.7	18.22	8.18	1.07
Conversion Price 17	1964	9.875	5.5	9.40	5.24	1.05
Converts to .5 Shares	1963	7.25	5.75	4.90	3.89	1.48

The five factors for determining the trend of the stock market were subjectively determined to be:

a. Expert opinion: value +1.

b. General economic conditions: value -1.

c. Specific economic conditions: value -1.

d. Stock market indices and data: value +1

e. Technical considerations: value +1

Since the sum of these factors is positive, the stock market trend is up.

The remainder of this section will consider the potential hedge situation candidates one at a time.

First National Realty

Initial common stock price: 2.5

Initial Warrant Price: 1.625.

Price range determination:

a. Average high price: 3 (From Equation 2)

b. Average low price: 1 (From Equation 3)

c. Average growth rate: -2.164 (From Equation 6)

d. Projected earnings: -.86 (From Equation 7)

e. Determined price range: 1 to 3 (From Page 29)

Adjusted conversion price: $6.75 \div 1.15 = 5.87$ (From Equation 9)

Figure 4 is the profit graph for this situation.

Short only intercept on the common price axis: $(1.625 + 6.75) \div 1.15 = 7.28$ (From Equation 10)

Since the entire price range lies to the left of the adjusted conversion

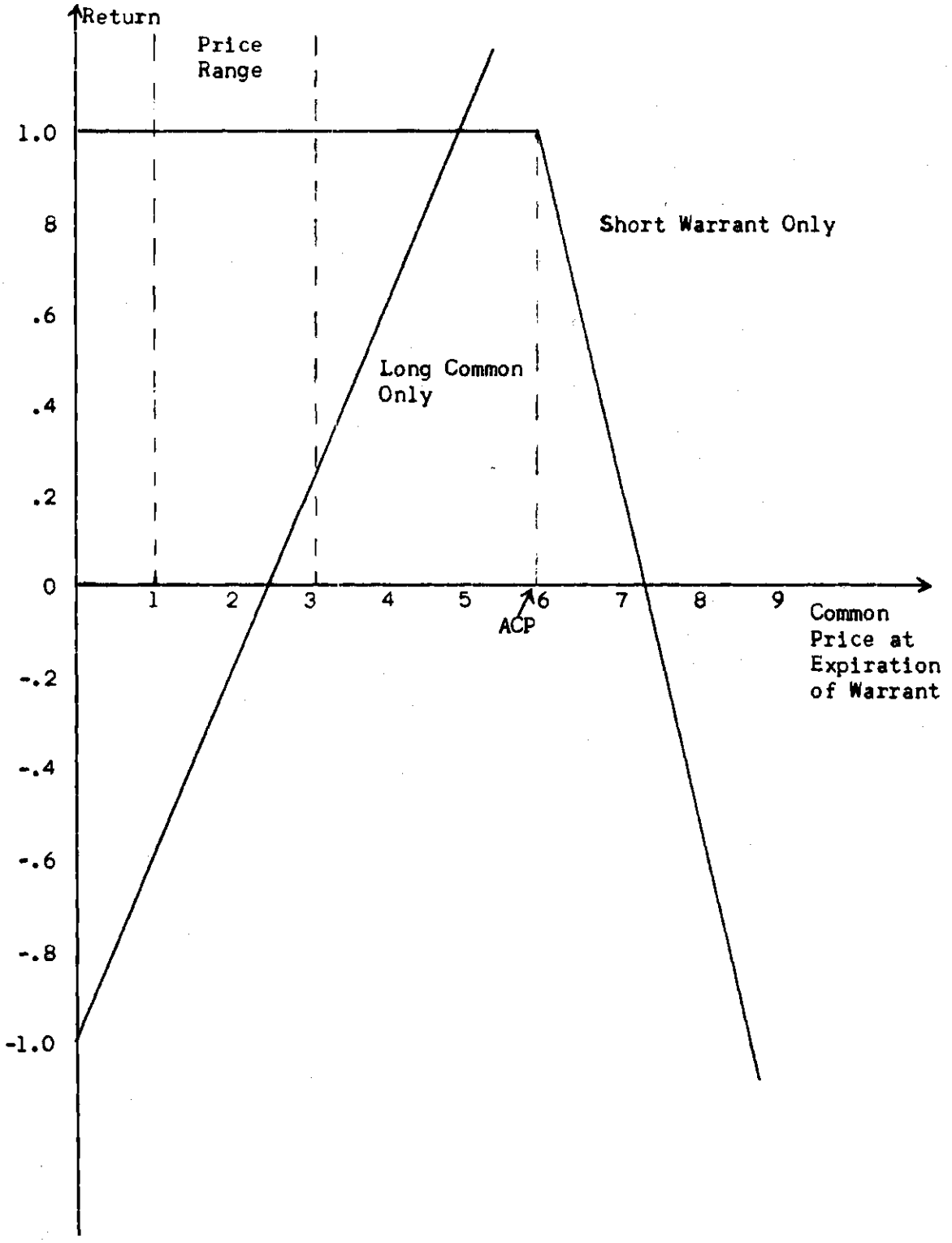

Figure 4. Profit Graph for First National Realty

price, a short warrant only position should be established as profit will be maximized as long as the common stock price remains in this price range.

Expected return: 1.0.

Current margin rate of interest: 6.25% per year.

Monthly margin rate of interest: .521% per month.

Number of months to expiration of warrant: 47.

Present Value: $1.0(1+.00521)^{-4} = .783$. (From Equation 21)

Martin Marietta

Initial common stock price: 19.625.

Initial warrant price: 15.75.

Price range determination:

a. Average high price: 25. (From Equation 2)

b. Average low price: 17. (From Equation 3)

c. Average high PE ratio: 14.7 (From Equation 4)

d. Average low PE ratio: 10.74 (From Equation 5)

e. Average growth rate: .016 (From Equation 6)

f. Projected earnings: 1.67 (From Equation 7)

g. Determined price range: 18 to 27. (From Page 29)

Adjusted conversion Price: $45 \div 2.73 = 16.49$ (From Equation 9)

Figure 5 is the profit graph for this situation.

Short only intercept on the common price axis: $(15.75+45) \div 2.73 = 22.25$ (From Equation 10)

Equation of short warrant only profit curve: $y = -.3356x + 6.976$. (From Equation 11)

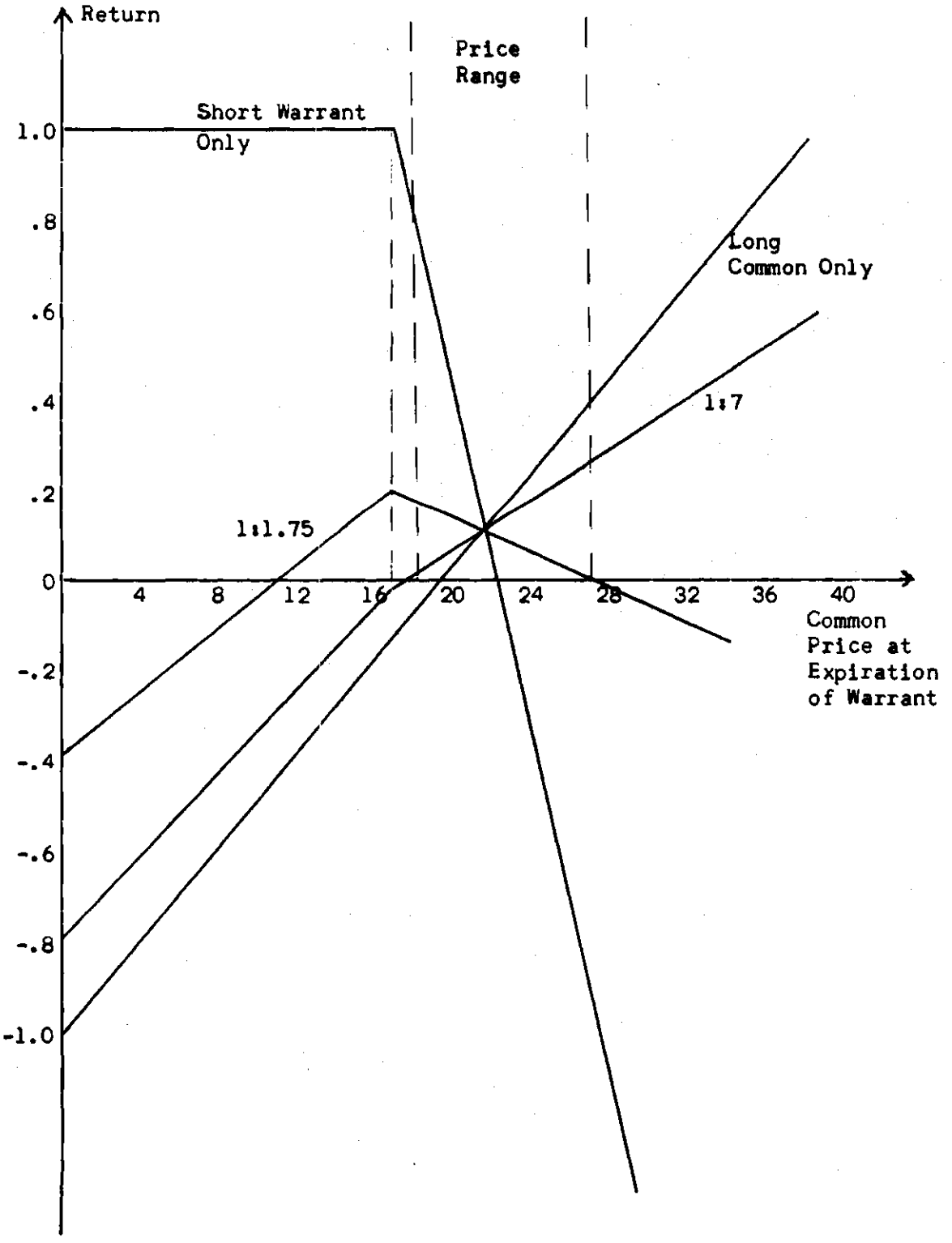

Figure 5. Profit Graph for Martin Marietta.

Equation of long common only profit curve: $y = .051x - 1$ (From Equation 12)

Intersection point of these curves: (20.166, .0285).

Hedge ratio whose profit curve crosses the common price axis at the lower price range limit:

$$(15.75 - (18(2.73) - 45))\ \text{Ratio} = 19.625 - 18$$

$$\text{Ratio} = .14 = \frac{1}{7} \qquad \text{(From Equation 17)}$$

Hedge ratio whose profit curve crosses the common price axis at the upper price range limit:

$$((27(2.73) - 45) - 15.75)\ \text{Ratio} = 27 - 19.625$$

$$\text{Ratio} = \frac{1}{1.75} \qquad \text{(From Equation 18)}$$

Table 2 tabulates the points required to plot the profit curves. Tabulation in the form of Table 2 is a convenient way to illustrate the required information for evaluating hedge situations. This type of tabulation will be used throughout the single decision and multi-decision evaluations. Under the first column of the table the three particular common stock prices necessary to construct the profit graph are listed. Column two indicates the theoretical warrant value, as determined from Equation (1), for each of these common stock prices. Column three lists the gain on the warrant at each of the common stock prices, if the warrant was selling at its theoretical value, as determined from Equation (13). Gains on the common stock at each of the particular common stock prices, as determined from Equation (14), are listed in column four. Columns five and six list the return at each of the common stock prices for the hedge ratios whose profit curves cross the price range limit points as determined previously. The value of M for each of these

ratios is listed above the table, and the return is computed from Equation (16)

Money required: R = 1:7, M = 15.75 + 19.625(7) = 153.13 (From Equation 15)

R = 1:1.75, M = 15.75 + 19.625(1.75) = 50.09

Table 2. Return at Various Common Stock Prices and Hedge Ratios for Martin Marietta

Common Price	Theoretical Warrant Value	Gain on Warrant	Gain on Common	R=1:7 Return	R=1:1.75 Return
0	0	15.75	-19.625	-.794	-.371
16.49	0	15.75	-3.135	-.04	.204
27	28.71	-12.96	9.085	.331	.0587

Figure 6 illustrates the estimated probability distribution for the price range of the common stock.

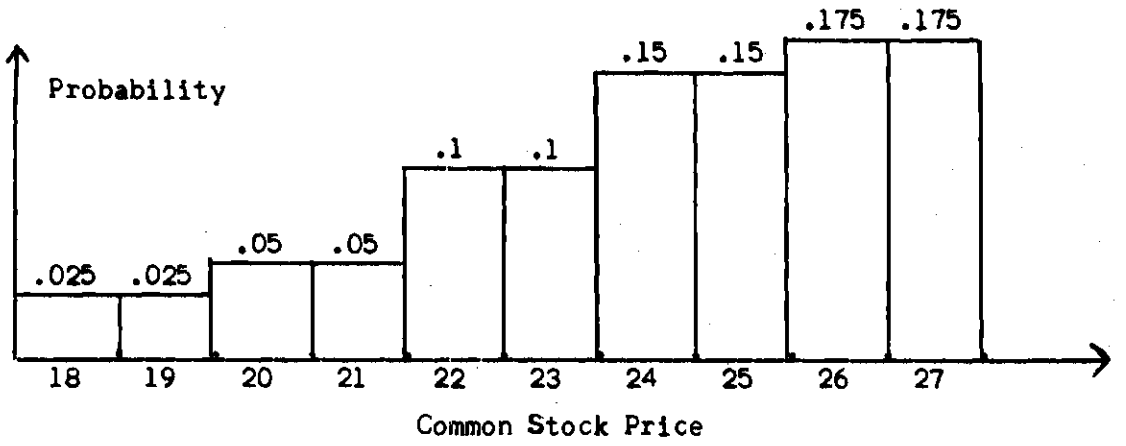

Figure 6. Assumed Probability Distribution of Martin Marietta.

Expected return (Ratio = 1:7): .178 (From Equation 19)

Expected return (Ratio = 1:1.75):.0558 (From Equation 19)

Select ratio 1:7 since this ratio yields the greatest expected return.

Current monthly margin rate of interest: .521%.

Number of months to expiration of warrant: 9.

Present value: $.178\ (1.00521)^{-9} = .169$ (From Equation 21)

Pacific Petroleum

Initial common stock price: 17.125.

Initial warrant price: 2.875.

Price range determination:

a. Average high price: 15. (From Equation 2)

b. Average low price: 9. (From Equation 3)

c. Average high PE ratio: 54.73 (From Equation 4)

d. Average low PE ratio: 36.86. (From Equation 5)

e. Average growth rate: .32 (From Equation 6)

f. Projected earnings: .70 (From Equation 7)

g. Determined price range: 13 to 26. (From page 29)

Adjusted conversion price: $19 \div 1.1 = 17.27$ (From Equation 9)

Figure 7 is the profit graph for this situation.

Short only intercept on the common price axis: $(2.875 + 19) \div 1.1 = 19.89$ (From Equation 10)

Equation of short warrant only profit curve: $y = -.382 + 7.6$. (From Equation 11)

Equation of long common only profit curve: $y = .0584x - 1$. (From Equation 12)

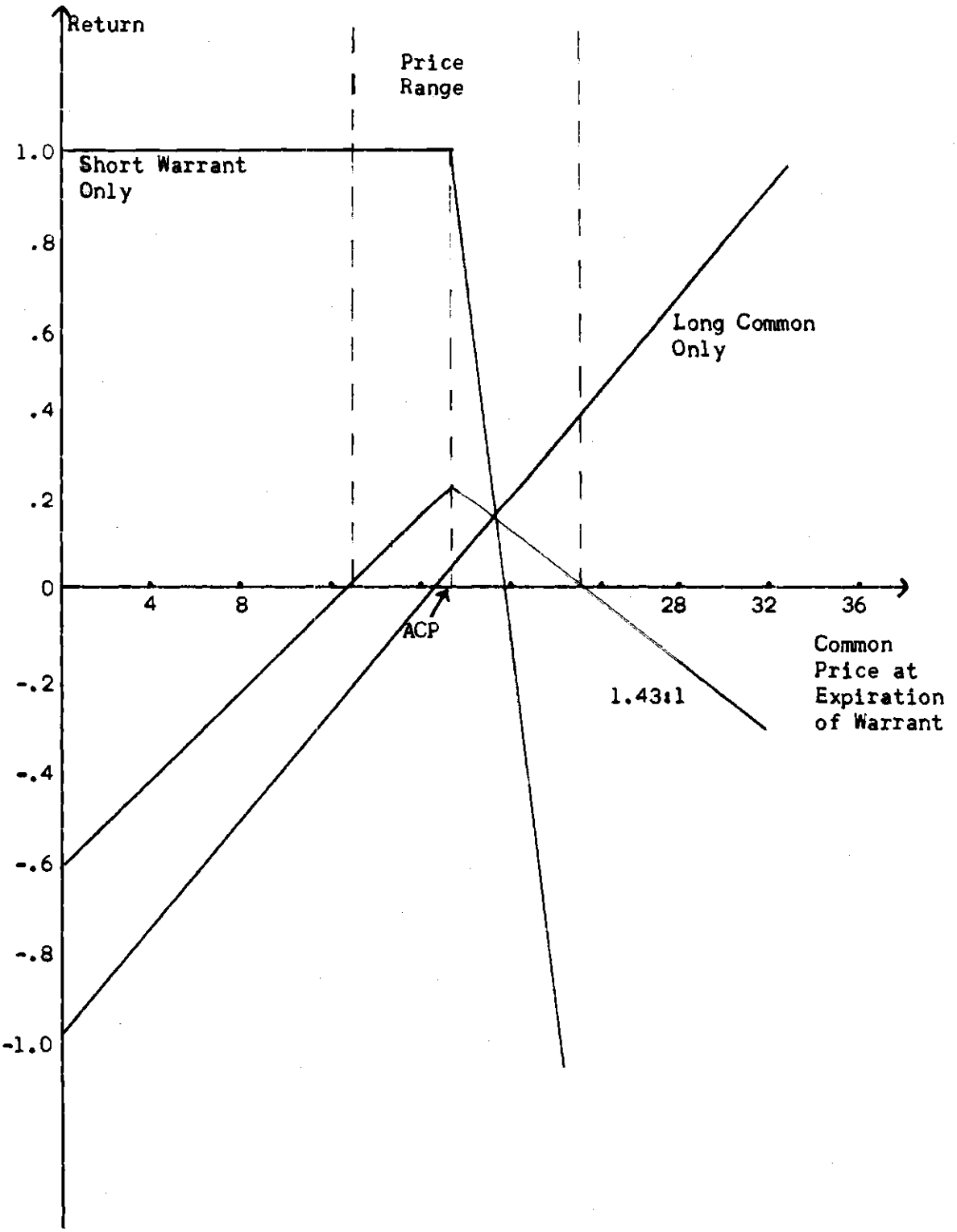

Figure 7. Profit Graph for Pacific Petroleum.

Intersection point of these curves: (19.52, .14)

Hedge ratio whose profit curve crosses the common price axis at the lower price range limit:

$$(2.875 - 0)\ \text{Ratio} = 17.125 - 13$$

$$\text{Ratio} = 1.43 \quad \text{(From Equation 17)}$$

Hedge ratio whose profit curve crosses the common price axis at the upper price range limit: Ratio = 1.32 (From Equation 18)

Table 3 tabulates the points required to plot the profit curves.

Money required: $R = 1.43{:}1$, $M = (2.875)(1.43) + 17.125 = 21.24$ (From Equation 15)

$R = 1.32{:}1$, $M = (2.875)(1.32) + 17.125 = 20.92$

Table 3. Return at Various Common Stock Prices and Hedge Ratios for Pacific Petroleum

Common Price	Theoretical Warrant Value	Gain on Warrant	Gain on Common	R=1.43:1 Return	R=1.32:1 Return
0	0	2.875	-17.125	-.613	-.637
17.27	0	2.875	.145	.20	.188
30	14	-11.125	12.875	-.143	.087
17.24	0	2.875	.115	.199	****

Inspection of Figure 7 indicates that none of the profit curves span the entire price range. Therefore, this situation should be eliminated from further consideration or the price range can be narrowed.

Since the warrant is less than two months from expiration, and based upon the estimated probability distribution, it was decided to narrow the range to incorporate the 1.43:1 ratio. The new range thus

becomes 13 to 23.

Figure 8 illustrates the estimated probability distribution for the new price range of the common stock.

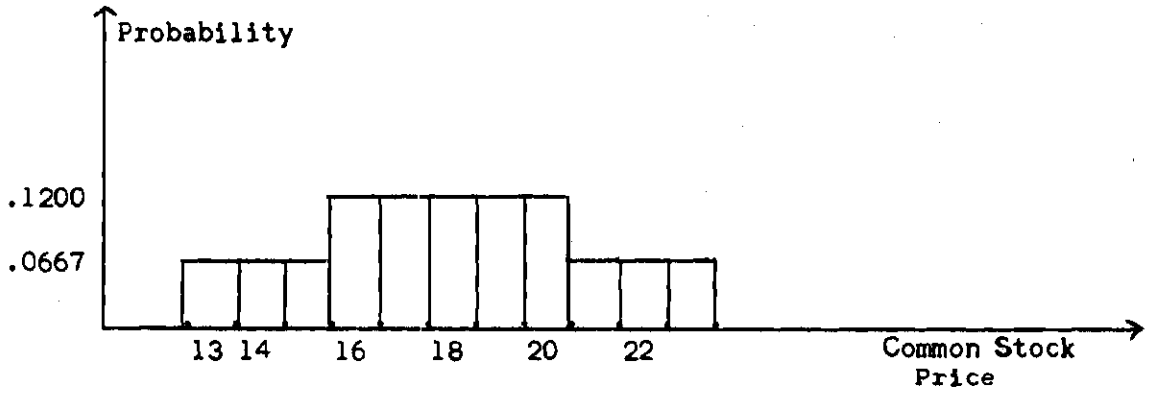

Figure 8. Assumed Probability Distribution of Pacific Petroleum

Since the hedge ratio of 1.43:1 is the only one that spans the entire price range it is selected in this situation.

Expected return: .119 (From Equation 19)

Current monthly margin rate of interest: .521%

Number of months to expiration of warrant: 2.

Present value: $.119\ (1.00521)^{-2} = .117$ (From Equation 21)

United Industrial Corporation

Initial common stock price: 15.

Initial warrant price: 3.25

Price range determination:

a. Average high price: 18. (From Equation 2)

b. Average low price: 8. (From Equation 3)

c. Average high PE ratio: 18.18 (From Equation 4)

d. Average low PE ratio: 9.64 (From Equation 5)

e. Average growth rate: .183 (From Equation 6)

f. Projected earnings: .44. (From Equation 7)

g. Determined price range: 8 to 18. (From page 29)

Adjusted conversion price: 17. ÷ .5 = 34 (From Equation 9)

Figure 9 is the profit graph for this situation.

Short only intercept on the common price axis: (3.25 +17) ÷ 5 = 40.5 (From Equation 10)

Since the entire price range lies to the left of the adjusted conversion price a short warrant only position should be established as profit will be maximized as long as the common stock price remains in this price range.

Expected return: 1.0

Current monthly margin rate of interest: .521%.

Number of months to expiration of the warrant: 22.

Present value: $1.0(1.00521)^{-22} = .892$. (From Equation 21)

With the preceding evaluations completed, the four potential hedge situations can be ranked in descending order of present value. This ranking is:

Corporation	Present Value
a. United Industrial Corp.	.892
b. First National Realty	.783
c. Martin Marietta	.169
d. Pacific Petroleum	.117

At this point, the investor can choose from these candidates the situation he desires to commit his resources to.

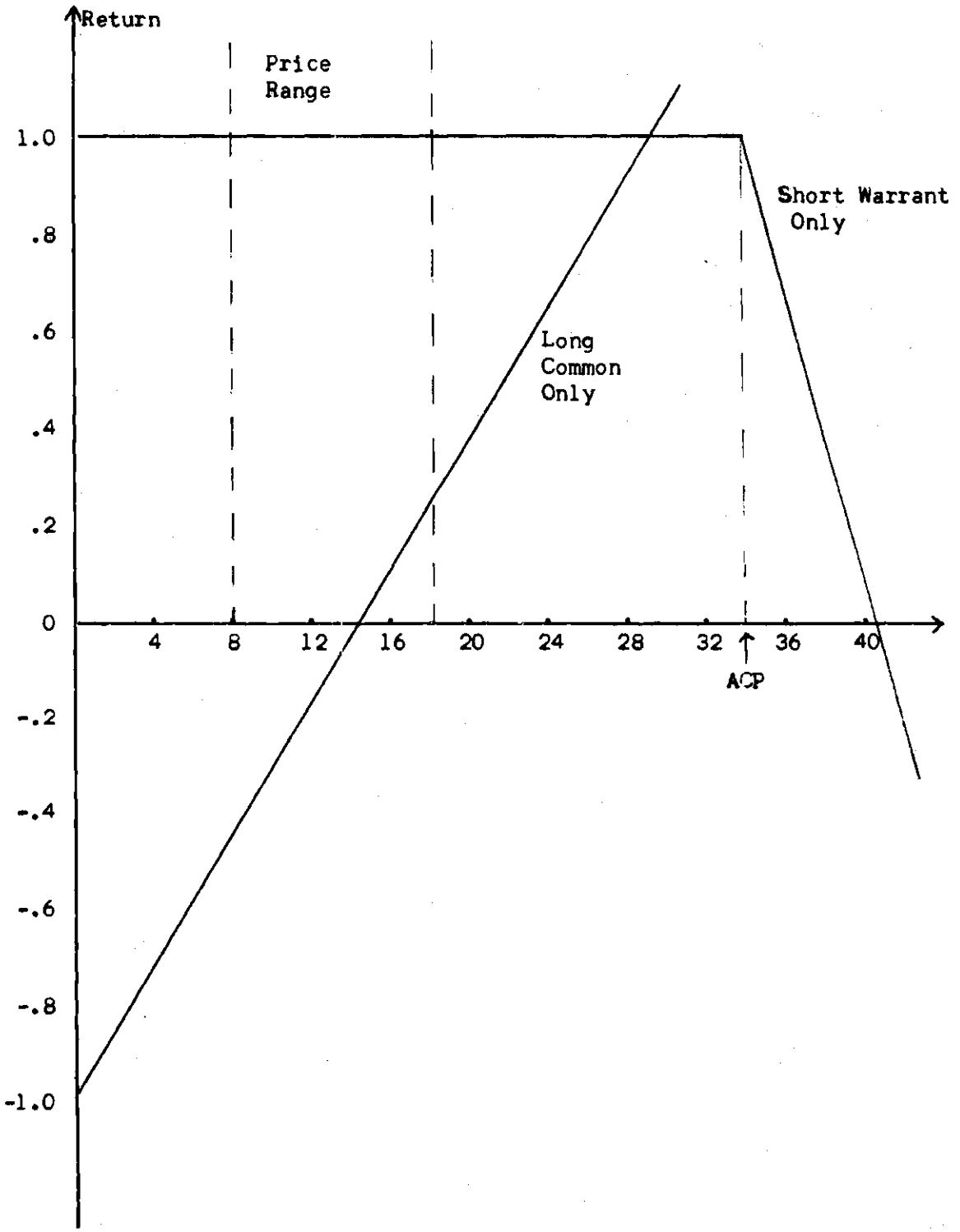

Figure 9. Profit Graph for United Industrial.

Dynamic Multi-Decision Process

Each of the above situations will be carried through this process as a means of completely illustrating the developed procedures and rules. The time interval selected to monitor each situation was four weeks. This seems to be sufficient to detect the various changes that may occur, yet it does not seem to require excessive investor time. The complete list of common stock and warrant prices from February 12, 1968 to December 31, 1971 is included in the Appendix.

United Industrial

Synopsis of pertinent information

a. Price range: 8 to 18.

b. Trend of market: up.

c. ACP: 34.

d. Current Position: Short warrant only.

Event No. 1: Upper price range exceeded on 6/3/68 at 19.75.

Using nine previous prices:

$$\bar{x} = \frac{151.375}{9} = 16.82$$

$$\sum_{i=1}^{9} (x_i - \bar{x})^2 = 45.883$$

$$\text{Standard Deviation} = \left(\frac{45.883}{8}\right)^{1/2} = 2.395 \quad \text{(From Equation 8)}$$

New upper price range limit: $19.75 + 3(2.395) = 27$

New lower price range limit: $19.75 - 3(2.395) = 12$

Range is still to the left of the adjusted conversion price therefore, continue short warrant only position.

Event No. 2: Trend of market changes in March 1969.

In retrospect, the trend of the market changed in late 1968 but was not detected by the author until March 1969.

Change price range using opposite procedures from 12 to 27 to 8 to 18. Current common stock price: 15.875.

This range remains to the left of the adjusted conversion price therefore, retain short warrant only position. No other events occurred.

Near the expiration date of the warrant:

a. Common stock price: 14.375

b. Approximate warrant price: 0

Actual return on investment: 1.0.

First National Realty

Synopsis of pertinent information:

a. Price range: 1 to 3.

b. Trend of market: up

c. ACP: 5.87

d. Current position: Short warrant only.

Event No. 1: Upper price range exceeded on 7/29/68 at 4.25.

Using 13 previous prices:

$$\bar{x} = 37.25 \div 13 = 2.87$$

$$\sum_{i=1}^{13} (x_i - \bar{x})^2 = 5.041$$

Standard Deviation $= \left(\frac{5.041}{12}\right)^{1/2} = .648$ (From Equation 8)

New upper price range limit: $4.25 + 3(.648) = 6.194 = 7$

New lower price range limit: $4.25 - 3(.648) = 2.306 = 2$

In this case it is not necessary to compute any hedge ratios since the upper price limit lies between the adjusted conversion price and the

short warrant only profit curve intercept on the common price axis. Therefore, the short warrant only position should be continued since this will yield maximum return within this price range.

Event No. 2: Upper price range exceeded on 12/2/68 at 8.875.

Using nine previous prices:

$$\bar{x} = 58 + 9 = 6.44$$

$$\sum_{i=1}^{9} (x_i - \bar{x})^2 = 17.231$$

$$\text{Standard Deviation} = \left(\frac{17.231}{8}\right)^{1/2} = 1.468 \quad \text{(From Equation 8)}$$

New upper price range limit: $8.875 + 3(1.468) = 13.279 = 14$

New lower price range limit: $8.875 - 3(1.468) = 4.471 = 4$

Determine the hedge ratio whose profit curve passes through the upper price range limit on the common price axis:

$$(9.35 - 6.375)\ \text{Ratio} = 14 - 8.875$$

$$\text{Ratio} = 1.72 \quad \text{(From Equation 18)}$$

Determine the hedge ratio whose profit curve passes through the lower price range limit on the common price axis:

$$(6.375 - 0)\ \text{Ratio} = 8.875 - 4$$

$$\text{Ratio} = .765 = \frac{1}{1.31} \quad \text{(From Equation 17)}$$

Close out previous position:

$$\text{Gain} = 1.625 - 6.375(1) = -4.75 \quad \text{(From Equation 22)}$$

Therefore a loss is incurred on this investment.

Money required:

$R = 1.72{:}1$, $M' = 6.375(1.72) + 8.875 + 4.75 = 24.59$

$R = 1{:}1.31$, $M' = 6.375 + 8.875(1.31) + 4.75 = 22.75$ (From Equation 23)

Table 4. Return at Various Common Stock Prices and Hedge Ratios for First National Realty

Common Price	Theoretical Warrant Value	Gain on Warrant	Gain on Common	R=1.72:1 Return	R=1:1.31 Return
4	0	6.375	-4.875	.248	0
5.87	0	6.375	-3.005	.324	.107
14	9.35	-2.975	5.125	0	.164

Figure 10 illustrates the estimated probability distribution for the new price range of the common stock.

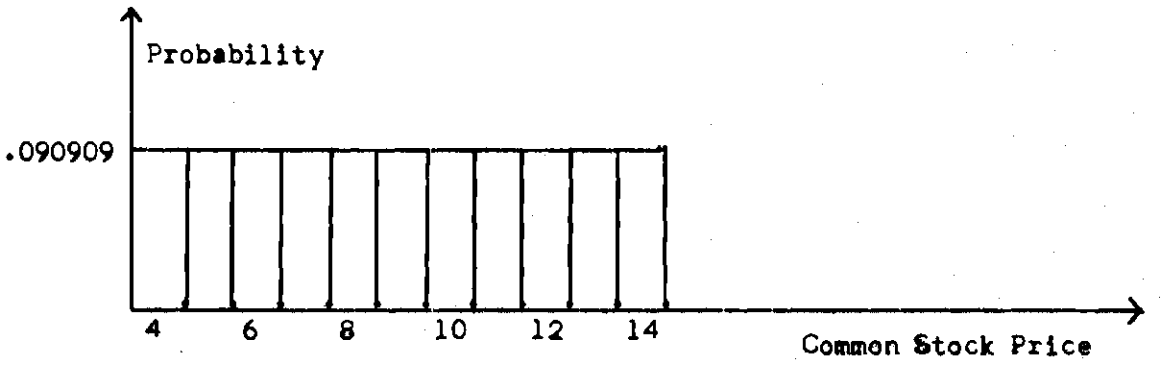

Figure 10. Assumed Probability Distribution of First National Realty

Expected return (Ratio = 1.72:1):.180 (From Equation 19)

Expected return (Ratio = 1:1.31):.116 (From Equation 19)

Figure 11 is the profit graph for this new situation.

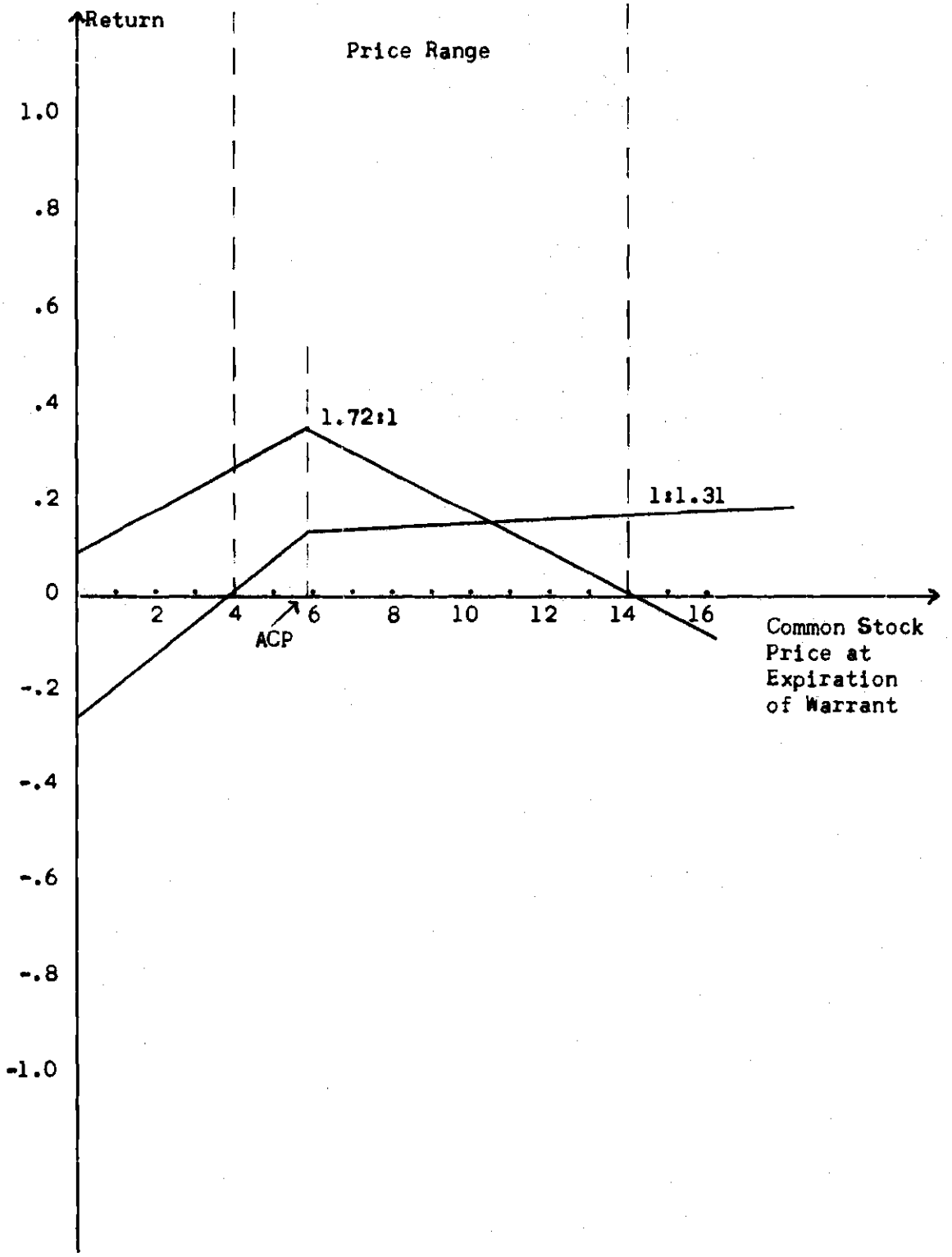

Figure 11. Revised Position Profit Graph for First National Realty.

The ratio 1.72:1 is selected, since its expected return is greatest.

Expected return: $\frac{(9(1.15) - 6.75)1.72 + 9 - 8.875}{24.59} = .257$

(From Equation 16)

Event No. 3: Lower price range exceeded 5/25/70 at 3.

Warrant price: 1.625.

Using 12 previous prices

$$\bar{x} = 62.125 \div 12 = 5.18$$

$$\sum_{i=1}^{12} (x_i - \bar{x})^2 = 13.222$$

Standard Deviation $= \left(\frac{13.222}{11}\right)^{1/2} = 1.097$ (From Equation 8)

New upper price range limit: $3 + 3(1.097) = 7$

New lower price range limit: $3 - 3(1.097) = 0$

The upper price range limit is to the left of the short warrant intercept on the common price axis of the profit graph, as previously determined, therefore, the most profitable position will be a short warrant only position.

Close out previous position:

Gain $= (3 - 8.875)1 + (6.375 - 1.625)1.72 = 2.295$ (From Equation 22)

Money required: Short only, $M' = 1.625 + 4.75 - 2.295 = 4.08$

(From Equation 23)

Figure 12 illustrates the estimated probability distribution for the new price range of the common stock.

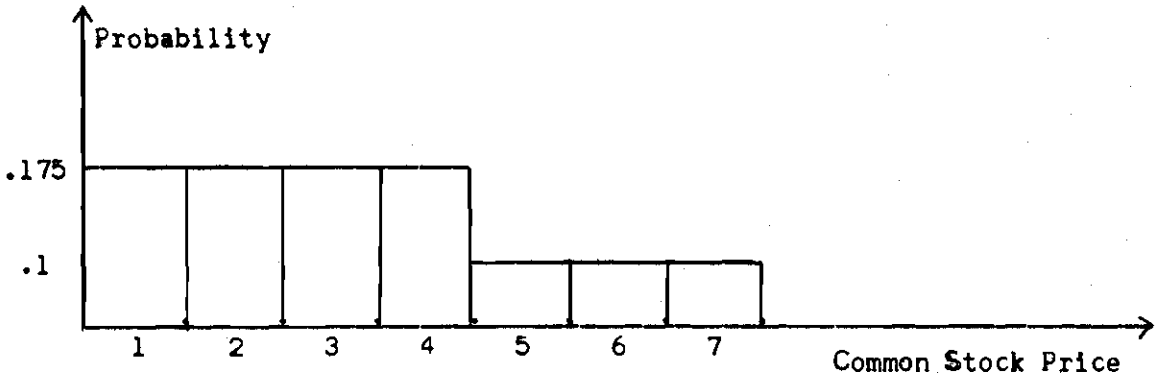

Figure 12. Assumed Probability Distribution of First National Realty.

Expected return: .363 (From Equation 19)

No other events occurred.

Near the expiration date of the warrant:

a. Common stock price: 1.375.

b. Approximate warrant price: 0

Actual return on investment: .398

Pacific Petroleum

Synopsis of pertinent information:

a. Price range 13 to 23.

b. Trend of market: up.

c. ACP: 17.27

d. Current position: Hedge ratio 1.43:1

No events occurred.

Near the expiration date of the warrant:

a. Common stock price: 15.75.

b. Approximate warrant price: 0

Actual return on investment: .129.

Martin Marietta

Synopsis of pertinent information:

a. Price range: 18 to 27.

b. Trend of market: up.

c. ACP: 16.49

d. Current position: Hedge ratio .14:1.

No events occurred.

Near the expiration date of the warrant:

a. Common stock price: 26.5

b. Approximate warrant price 27.375.

Actual return on investment: .238

Summary

The results of using a stock market hedge situation are quite favorable. In capsule form the total return of each of the illustrative examples are as follows:

a. United Industrial: 1.0

b. First National Realty: .398

c. Martin Marietta: .238

d. Pacific Petroleum: .129

In considering the time value of money, assuming all cash to commit to these various situations was borrowed at the rate of 6.25% per annum, the returns are:

a. United Industrial: .892

b. First National Realty: .311.

c. Martin Marietta: .227.

d. Pacific Petroleum: .128.

One final comparison is to examine the present value as determined from the expected return of the single decision process against the actual present value after applying the dynamic multi-decision process. These results are shown below:

Corporation	Single Decision Present Value	Actual Present Value
a. United Industrial	.892	.892
b. First Nat'l Realty	.783	.311
c. Martin Marietta	.169	.227
d. Pacific Petroleum	.117	.128

The only one of these situations that actually had any changing of position during the course of the dynamic process was First National Realty. United Industrial was a short warrant only commitment. The final price of the common stock was less than the adjusted conversion price consequently, the present value was .892 as long as the common stock was less than $34 per share. This wide price range was one in which a probability distribution was virtually not required. In the case of Martin Marietta and Pacific Petroleum, the reason for the discrepancy between the two present values is because the original probability distribution was erroneous. In the case of First National Realty, the final result would have been a present value of .783 had no changes been made during the dynamic process. This is because the final common stock price was very close to the original common price when the warrant expired. In this situation the stock made a complete circle in price. The results here do not destroy the validity of the dynamic multi-decision process. In December, 1968 and January, 1969 when the common

stock was trading in the 11 - 12 range, there was little to indicate that the stock would reverse its course as drastically as it ultimately did. On the contrary, it would have taken a steel-nerved investor to continue to hold a short warrant only position established when the common stock was at 2.5.

In the final analysis, the results actually obtained by applying the rules and procedures developed in this thesis indicate the potential of these processes in this type of investment.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

As stated by Markowitz (20) there are two objectives common to all investors. The first objective is that investors want the return on their investment to be high. Secondly, they want this return to be dependable and not subject to uncertainty. The rules and procedures developed in this thesis for the stock market hedge situation seem to satisfy, to a great degree, these objectives.

An investor desiring to invest following these rules and procedures is generally going to be one who is willing to trade off possible profits for a greatly reduced risk in incurring a loss on his investment. The illustrative examples presented in Chapter V clearly show that profits, substantial profits, can be obtained by using the processes developed. There is nothing to indicate that similar situations cannot be found in today's stock market.

The extensive use of the profit graph in this thesis shows, in graphical form, the various positions that can be established along with the common stock price range that will insure profitability for each position. This is the first known formal presentation of these varying positions in graphical form. By using this graphical aid the investor can easily determine his alternatives for selection of commitment positions without excessive difficulty.

In comparing this work with the work previously done by Thorp and Kassouf (33), the Martin Marietta and United Industrial situations indicate the advantage of using the specific decision rules developed in this thesis. Thorp and Kassouf recommended that the Martin Marietta and United Industrial situations be eliminated from consideration as these common stocks were selling at more than 1.2 times their respective adjusted conversion prices. This figure of 1.2 was an unsubstantiated guideline they mentioned in their work. It is shown in Chapter V however, that this guideline is not necessarily valid when using the rules and procedures developed in this study as both of these situations yielded substantial returns.

It is difficult to find a similar system with which to compare the rules and procedures developed in this thesis. Since no other authors have taken the approach of outlining specific decision rules for the hedge situation it is felt that the rules and procedures developed in this study are superior to the other approaches in that there is substantiation for each rule and procedure. Furthermore, each example in Chapter V, following the developed rules and procedures, was profitable.

If the majority of investors in the securities market practiced the hedge situation investment as described in this thesis, the rules and procedures could lose much of their validity as conditions in the stock market affecting warrant prices would likely change. Since this event is not considered very likely, it must be concluded that the system developed in this thesis should have a bright future. As previously shown, the rules and procedures do work. The number of warrants trading

on the NYSE and the ASE have increased significantly, now totaling 43, thus affording the investor many more potential hedge situation candidates from which to choose. In addition, if consideration is given to warrants trading on other regional exchanges or in the over-the-counter market another 259 situations are available.

An obvious conclusion to this thesis is the fact that most of the rules and procedures could easily be computerized. It has been intentionally structured to facilitate computerization. This would significantly reduce the time required to evaluate the various situations.

Limitations of Research

There are two primary problem areas that have a limiting effect on this study. The first is that the longer period of time a warrant has until it expires, the more difficult it is to accurately determine a common stock price range and to assign a probability distribution over this price range. This is an obvious problem as many unknown and unforseeable events can occur over time that can easily affect the final common stock price when the warrant expires. The First National Realty example in Chapter V shows this quite clearly.

The greatest problem encountered in this study was in assessing probability distributions over the common stock price range. As shown in Chapter V, erroneous probability distributions largely resulted in the discrepancies between expected return and actual return. These problem areas are recommended for future research in the next section of this chapter.

Recommendations

The investigation of methods for selecting and controlling investment alternatives, within the framework of the stock market hedge situation, has provided insights into areas that are important for future research. This study can provide the basis for much of this future research as many of these areas are logical extensions of this work. Some of the areas to be included were considered but determined to be outside the present scope of study.

The primary area of possible future research is in the direction of the Markowitz portfolio selection model. This thesis takes the approach that investment risk is reduced through a widening of the common stock price range. It further is oriented toward the selection of one hedge situation candidate from a group of possible candidates. The work by Markowitz attempts to quantify risk and return through the use of efficient portfolios which consist of either a set of stocks which have a maximum expectation of return for a given variance in the return, or alternatively, a set of stocks which for a fixed return, have a minimum variance in returns. The work by Markowitz represents a more statistically sophisticated approach and is a very logical area to explore.

In a similar context, current work in capital budgeting could be incorporated into this study. As mentioned, this thesis is primarily concerned with the selection of one investment situation from a group of potential situations. Additional research to determine optimal allocation policies through the use of varying program approaches applied to the group of potential situations is also a logical area to explore.

As mentioned in the previous section, probability assessment is

one of the major problems encountered in this study. More research and extension in this area could produce fruitful results. This work could logically include subjective assessment of discrete probability distributions as well as assessment of known continuous probability distributions with varying parameters. In connection with this future probability study, techniques for various sensitivity analyses could be developed in attempts to determine just how varying of probability distributions will affect the expected return.

A purely analytical approach to the problems treated in this study could be accomplished. For example, equations could be determined for the various profit curves and decision solutions could be obtained analytically. This type of approach is much more general in nature and would most likely require use of continuous probability distributions. While this type of an approach would be difficult, it does appear to be feasible and is worthy of future consideration.

Another recommendation is to prepare a computer program, possibly incorporating some of the previous recommendations, to make application of the decision rules and procedures a more efficient, less time consuming process.

A final recommendation is to investigate the application of utility theory to this problem. Through this extension, risk preferences for individual investors could be considered. While this approach was not considered in this thesis, sensitivity analysis could be used to study the impact of various utility functions on the expected return. That is, would risk seeking or risk averse investors perform significantly different in the hedge situation.

APPENDIX

Common Stock and Warrant Prices from Feb. 12, 1968 to Dec. 31, 1971

	First Nat'l Realty		Martin Marietta		Pacific Petroleum		United Industrial	
Date	Common	Warrant	Common	Warrant	Common	Warrant	Common	Warrant
2/12/68	2.5	1.625	19.625	15.75	17.125	2.875	15.0	3.25
2/26/68	2.375	1.5	19.0	15.25	16.875	1.375	14.625	3.125
3/11/68	2.125	1.062	18.75	13.25	15.5	1.0	15.5	3.125
3/25/68	2.125	1.125	18.25	12.75	15.75	.062	14.0	2.75
4/08/68	2.5	1.25	19.625	14.875	*	*	16.0	3.0
4/22/68	2.5	1.25	20.125	16.0			17.125	3.125
5/06/68	2.5	1.25	21.875	17.5			18.625	3.75
5/20/68	2.75	1.5	22.875	21.5			20.75	4.25
6/03/68	3.375	1.75	23.25	21.375			19.75	4.625
6/17/68	3.625	1.75	23.25	22.125			20.75	4.25
7/01/68	3.375	1.625	21.875	19.75			20.25	4.625
7/15/68	3.25	1.5	22.25	20.0			22.375	5.25
7/29/68	4.25	2.625	21.5	16.5			20.75	4.625
8/12/68	4.125	2.375	21.5	16.5			19.375	4.5
8/26/68	4.75	3.25	25.0	22.5			18.25	4.25
9/09/68	5.75	3.75	24.375	22.25			18.875	4.5
9/23/68	6.875	5.0	27.125	28.5			17.875	3.875
10/07/68	7.75	6.125	29.5	35.25			19.125	4.0
10/21/68	7.25	5.75	26.5	27.25			17.625	4.0
11/04/68	6.375	4.625	*	*			17.25	3.625
11/18/68	6.25	4.625					16.625	3.375
12/02/68	8.875	6.375					18.5	4.0
12/16/68	11.5	8.5					17.625	3.75
12/30/68	12.25	9.25					17.125	3.75
1/13/69	11.25	9.0					18.0	3.875
2/10/69	11.0	8.875					18.75	3.875
3/10/69	9.5	7.125					15.875	3.0
4/07/69	10.125	7.75					15.625	2.5
5/05/69	9.75	7.25					12.875	2.125
6/02/69	9.25	7.375					16.375	2.125
6/30/69	7.5	5.25					13.0	1.25
7/28/69	5.75	4.0					12.875	1.375
8/25/69	7.25	5.25					14.75	1.625
9/22/69	6.0	4.375					14.0	.625
10/20/69	6.0	4.25					14.25	.312
11/10/69	6.0	3.875					14.375	.031
12/08/69	4.625	2.875					*	*

*Indicates warrant expired.

Common Stock and Warrant Prices from Feb. 12, 1968 to Dec. 31, 1971
(Continued)

	First Nat'l Realty	
Date	Common	Warrant
1/05/70	5.0	3.25
2/02/70	4.125	2.75
3/02/70	5.125	3.0
3/30/70	5.25	2.75
4/27/70	4.0	2.0
5/25/70	3.0	1.625
6/22/70	3.5	1.75
7/20/70	2.625	1.375
8/17/70	2.375	1.25
9/14/70	2.875	1.5
10/12/70	3.0	1.625
11/09/70	2.125	1.125
12/07/70	2.125	.875
1/04/71	2.0	.875
2/01/71	2.625	1.125
3/01/71	3.375	1.5
3/29/71	2.875	1.25
4/26/71	2.625	1.0
5/24/71	2.625	1.0
6/21/71	2.375	.75
10/11/71	1.75	.438
11/08/71	1.25	.062
12/27/71	1.355	.015

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